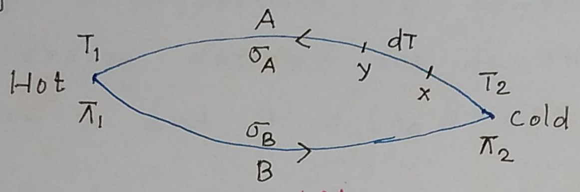
Topic → Current Electricity

Ques :- Show that in a thermocouple the total e.m.f. is given by

$$E = \pi_2 - \pi_1 + \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\, dT$$

Where the symbols have their usual meanings.

Let us consider a thermocouple consisting of two metals A and B having its hot junction at the absolute temperature T_1 and the cold junction at the absolute temperature T_2 as shown in fig.



(fig.)

Let σ_A be the Thomson co-efficient for the metal A and σ_B that for metal B, both considered to be positive. The thermo-electric current flows through the metal A from cold junction and through B from the hot junction thus completing the circuit. The Peltier co-efficient at temperature T_1 is π_1 and at temperature T_2 is π_2. If a current of I amperes passes through the circuit for t seconds, then

Energy absorbed due to Peltier effect at the hot junction $= \pi_1 I t$ Joule ——— (1)

Energy evolved at the cold junction due to Peltier effect $= \pi_2 \, It$ Joules. ——— (2)

Due to Thomson effect energy is absorbed in the metal A in which the current flows from lower temperature to higher temperature and evolved in the metal B in which current flows from higher temperature to a lower temperature.

Let us consider two points x and y having a difference of temperature dT, then heat energy absorbed due to Thomson effect between x and y $= (\sigma_A \, dT)\, It$.

$\therefore$ Total energy absorbed in going from T_2 to T_1 $= \left(\int_{T_2}^{T_1} \sigma_A \, dT \right) It$ ——— (3)

Similarly total heat energy evolved in the metal B is $= \left(\int_{T_1}^{T_2} \sigma_A \, dT \right) It$ ——— (4)

$\therefore$ Total gain in energy for the complete circuit is

$$= \pi_1 It - \pi It + \int_{T_2}^{T_1} \sigma_A \, dT \, It + \int_{T_1}^{T_2} \sigma_B \, dT \, It$$

$$= \left[\pi_1 - \pi_2 - \int_{T_1}^{T_2} \sigma_A \, dT + \int_{T_1}^{T_2} \sigma_B \, dT \right] It \quad \text{—} (5)$$

This energy appears as heat due to the thermo e.m.f. developed in the circuit. If E be the total e.m.f. produced in the circuit, then

Total gain in energy $= EIt$ ~~Joules~~ — (6)

$$\therefore \quad EIt = \left[\pi_1 - \pi_2 - \int_{T_1}^{T_2} \sigma_A \, dT + \int_{T_1}^{T_2} \sigma_B \, dT \right] It$$

or

$$E = (\pi_1 - \pi_2) - \int_{T_1}^{T_2} (\sigma_A - \sigma_B) \, dT \quad \text{—} (6)$$

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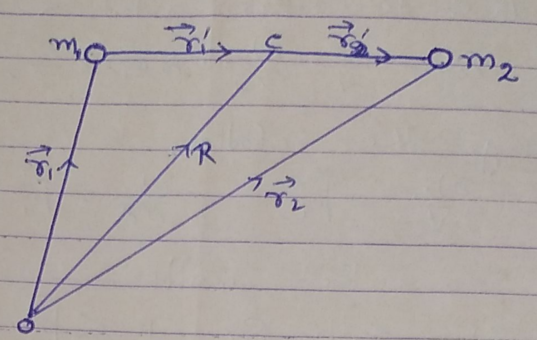
Two body Problem.



Let us consider a motion of a system ~~consi~~ consisting of two interacting particles of masses m_1 and m_2 whose radius vectors from the origin o are $\vec{r}_1$ & $\vec{r}_2$ respectively. The potential energy of interaction between two particles depends only on the distance between them. ~~The~~ The Lagrangian of the system is,

$$L = \frac{1}{2} m_1 \dot{r}_1^2 + \frac{1}{2} m_2 \dot{r}_2^2 - U(|\vec{r}_1 - \vec{r}_2|) \quad \text{—(1)}$$

where,

~~r~~ $\vec{r} = \vec{r}_1 - \vec{r}_2$ is the relative positive vector. Let the centre of mass of the system be at the point c, the radius vector of which from the origin is $\vec{R}$. Let the radius vectors of m_1 and m_2 w.r.t. c are $\vec{r}_1'$ and $\vec{r}_2'$ respectively. Thus we have,

$$m_1 \vec{r}_1' + m_2 \vec{r}_2' = 0 \quad \text{—(2)}$$

and,

$$\left.\begin{aligned} \vec{r}_1 &= \vec{R} + \vec{r}_1' \\ \vec{r}_2 &= \vec{R} + \vec{r}_2' \end{aligned}\right\} \quad \text{—(3)}$$

So that, $\vec{r}_1' - \vec{r}_2' = \vec{r}_1 - \vec{r}_2 = r$.

From ② we have,

$$(m_1+m_2)\vec{r}_1' = m_2(\vec{r}_1' - \vec{r}_2') = m_2\vec{r}.$$

$$\left.\begin{aligned} \text{or, } \vec{r}_1' &= \frac{m_2}{m_1+m_2}\vec{r} \\ \vec{r}_2' &= \vec{r}_1' - \vec{r} = -\frac{m_1}{m_1+m_2}\vec{r} \end{aligned}\right\} \quad \text{— ④}$$

The Kinetic energy of the system is,

$$T = \frac{1}{2}m_1\dot{r}_1^2 + \frac{1}{2}m_2\dot{r}_2^2$$

$$= \frac{1}{2}m_1(\dot{\vec{R}}+\dot{\vec{r}}_1').(\dot{\vec{R}}+\dot{\vec{r}}_1') + \frac{1}{2}m_2(\dot{\vec{R}}+\dot{\vec{r}}_2').(\dot{\vec{R}}.\dot{\vec{r}}_2').$$

$$= \frac{1}{2}(m_1+m_2)\dot{R}^2 + \frac{1}{2}(m_1\dot{r}_1'^2 + m_2\dot{r}_2'^2) + \dot{\vec{R}}.(m_1\dot{\vec{r}}_1' + m_2\dot{\vec{r}}_2') \quad \text{— ⑤}$$

With the help of ② we have,

$m_1\dot{\vec{r}}_1' + m_2\dot{\vec{r}}_2' = 0$ and ⑤ can be written as,

$$T = \frac{1}{2}(m_1+m_2)\dot{R}^2 + \frac{1}{2}\left[m_1\dot{r}_1'^2 + m_2\dot{r}_2'^2\right] \quad \text{— ⑥}$$

Using ④ it can be written as,

$$T = \frac{1}{2}(m_1+m_2)\dot{R}^2 + \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)\dot{r}^2$$

$$= \frac{1}{2}M\dot{R}^2 + \frac{1}{2}m\dot{r}^2 \quad \text{—— ⑦}$$

where, $M = m_1+m_2$ is the total mass of the system and

$m = \dfrac{m_1m_2}{m_1+m_2}$ is the ~~rep~~ reduced mass of the system.

Now the Lagrangian of the system is given by,

$$L = \frac{1}{2}M\dot{R}^2 + \frac{1}{2}m\dot{r}^2 - U(r) \quad \text{—— ⑧}$$

If the origin O be shifted at the centre C, then $R=0$ and ⑧