

Any n independent variables

q_1, q_2 In which correctly define the position of the system are called generalised co-ordinates of the system. These generalised co-ordinates may be cartesian co-ordinates, spherical polar co-ordinates,

The time derivatives of generalised co-ordinates q_i are called generalised velocities $\dot{q}_i$ ($i = 1, 2, 3, \dots, n$)

Generalised momentum:-

The position vector $\vec{r}_i$ is a vector function of generalised co-ordinates $q_1, q_2, q_3, \dots, q_n$ i.e.

$$\vec{r}_i \equiv \vec{r}_i(q_1, q_2, \dots, q_n) \dots \text{--- (1)}$$

The momentum of the system can be written as

$$\vec{p} = \sum_i m_i \vec{v}_i \dots \text{--- (2)}$$

From (1)

$$\vec{v}_i = \sum_{j=1}^n \frac{d\vec{r}_i}{dq_j} \dot{q}_j$$

$$\text{Hence, } \vec{v}_i = \frac{d\vec{r}_i}{dt} = \sum_{j=1}^n \frac{d\vec{r}_i}{dq_j} \frac{dq_j}{dt}$$

$$= \sum_j \frac{d\vec{r}_i}{dq_j} \dot{q}_j \dots \text{--- (3)}$$

Generalised Co-ordinates:-

In order to locate the position of a particle we may use Cartesian co-ordinates x, y, z . Hence it has three degree of freedom. If a system contains N particles, then it is necessary to have $3N$ co-ordinates. Hence the system has $3N$ degree of freedom.

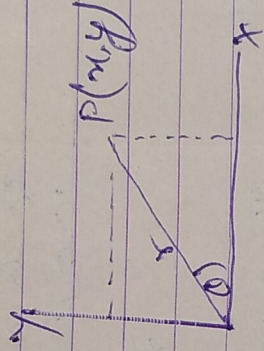
The choice of co-ordinates depends on the condition of the problem. For example for a particle moving in a central force in a plane, it is more convenient to use plane polar co-ordinates (r, θ) . They are related to Cartesian co-ordinates (x, y)

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\text{or } r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right)$$



Similarly we may use spherical polar co-ordinates (r, θ, ϕ) where ~~for~~ force is symmetric. Thus we use different type of co-ordinates for different types of problem. The choice is governed by the condition of problem concerned. However the number of co-ordinates used must be sufficient in number to define the state of a system correctly.