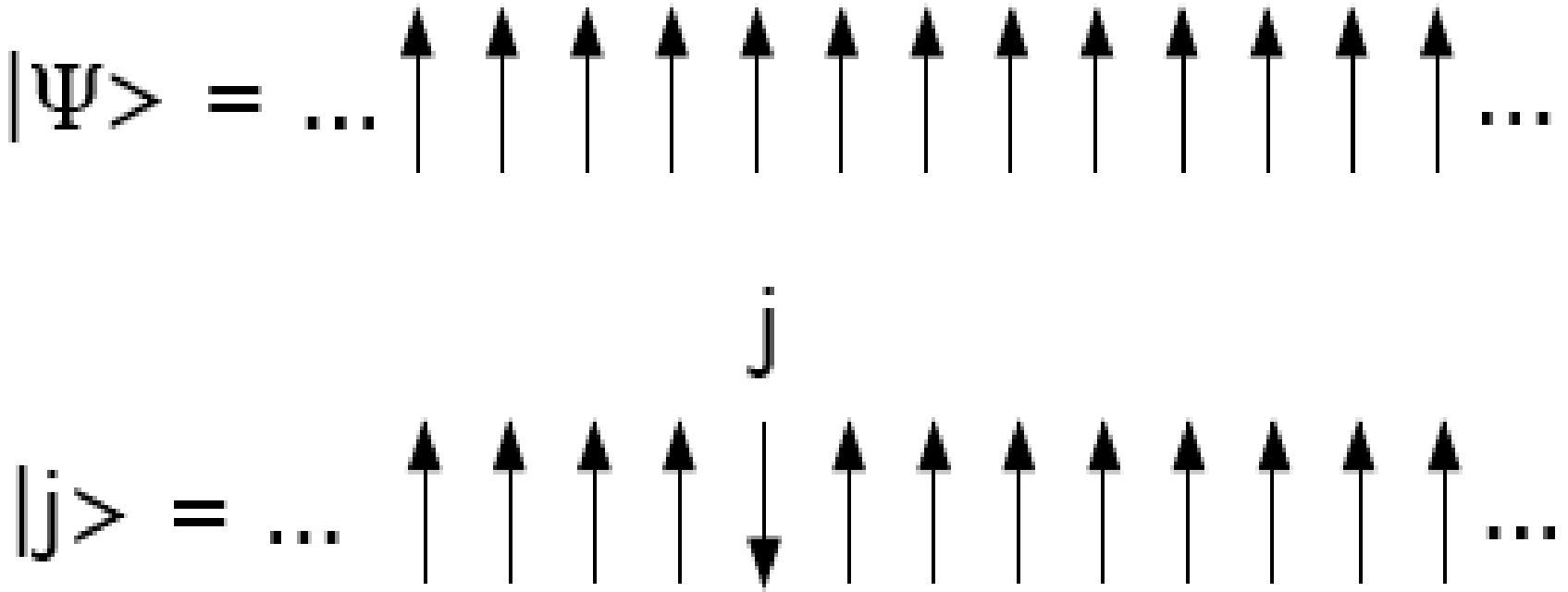
Quantisation of spin waves

In the ferromagnetic ground state , the spin waves are not excited and therefore all spins are parallel. The total spin quantum number of a system of N-spins , each S , in this state , has the value NS . The excitation of spin wave lowers the total spin value is reduced below the value NS .

Quantum mechanical derivation

Heisenberg Hamiltonian for a linear chain of spins

$$\hat{H} = -2J \sum_j \left[\hat{S}_i^z \hat{S}_{i+1}^z + \frac{1}{2} \left(\hat{S}_i^+ \hat{S}_{i+1}^- + \hat{S}_i^- \hat{S}_{i+1}^+ \right) \right]$$


|Ψ> = ...
...
j
|j> = ...
...

Ground state $|\emptyset\rangle$

$E_0 = NS_2J$

eigenvalue of the ground state $|\emptyset\rangle$

$|j\rangle$ excited state: spin flip at site j

$$|j\rangle = \hat{S}_j^- |\Psi\rangle$$

$$\hat{H}|\Psi\rangle = -NS^2 J|\Psi\rangle$$

ÄS=1 total change of the spin Magnons are Bosons!

Notice that |j > is not an eigenstate of the Hamiltonian because:

$$\hat{H}|j\rangle = 2\left[\left(-NS^2J + 2SJ\right)|j\rangle - SJ|j+1\rangle - SJ|j-1\rangle\right]$$

$$\hat{H}|j\rangle \neq c|j\rangle$$