It follows from these properties that any vector 'a' of linear space can be expanded in terms of complete set of base vectors;

$$a = \sum_{i=1}^{n} a^i \vec{\alpha}_i$$

where complex no. a^i is given by

$a^i = (\vec{\alpha}_i, a)$, which is defined as a component of a in this base.

For instance, in real 3-dimensional space the orthonormal basis consists of three mutually orthogonal vectors, each of unit length

i.e.,

$$i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad j = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad k = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and vector 'a' in this space is written as

$$\mathbf{a} = a_x \hat{i} + a_y j + a_z k,$$

with

$$a_x = \mathbf{a} \cdot i \; ; \; a_y = \mathbf{a} \cdot j \; , \; a_z = \mathbf{a} \cdot k$$

In another basis the components have the form

$$a'_x = \mathbf{a} \cdot i' \; ; \; a'_y = \mathbf{a} \cdot j' , \; a'_z = \mathbf{a} \cdot k'$$

with

$$|\mathbf{a}| = (a_x^2 + a_y^2 + a_z^2)^{1/2} = (a_x'^2 + a_y'^2 + a_z'^2)^{1/2}$$

Such basis invariant quantities are known as scalars. In n-dimensional linear space, a vector 'a' may be displayed in terms of its components a as,

$$a = \begin{pmatrix} a^1 \\ a^2 \\ \vdots \\ a^n \end{pmatrix}$$

where its direction and magnitude are fixed but its components vary with the basis. The representation of vector requires the basis in which its components are prescribed. The adjoint of the column vector, given by ~~eq~~ above eq., is a row vector with the elements as complex conjugates of corresponding elements of 'a',

i.e $\quad \text{adj } a = (a^{1*}, a^{2*}, \ldots a^{n*}) = a^{\dagger}$

In Dirac notation the above top eq. is written as

$$a = |a\rangle$$

where $|a\rangle$ is defined as ket vector.

Similarly $\quad \text{adj } a = \langle a|$

where $\langle a|$ is a bra vector. Then inner product

$$b.a = \langle b|a\rangle \text{ (bracket)}$$

$$= \sum_{i=1}^{n} b^{i*} a^{i}$$

which shows $\quad \langle b|a\rangle = \langle a|b\rangle^{*}$

i.e the dot product is nolonger a commutative operator for complex vectors.

Furthermore, we have

$$|a\rangle^{\dagger} = \langle a| \quad \text{and} \quad \langle a|^{\dagger} = |a\rangle^{*}$$

and $$(\langle b|a\rangle)^\dagger = |a\rangle^\dagger \langle b|^\dagger = \langle a|b\rangle = \langle b|a\rangle^*$$

In Dirac's notation eq. ~~...~~ $\vec{a} = \sum_{i=1}^{n} a^i \vec{\alpha}_i$ is written

$$|a\rangle = \sum_{i=1}^{n} a^i |\alpha_i\rangle$$

and

$$\langle a| = \sum_{i=1}^{n} \langle \alpha_i| a^{i*}$$

and the orthonormality relation ~~...~~ $(\vec{\alpha}_i, \vec{\alpha}_j) = \delta_{ij}$, $i,j = 1, \ldots n$ may then be written as

$$a^i = \langle \alpha_i | a \rangle,$$

Notes

Appointment

Contacts