Ex. 12.4
Q. 5

16/04/2021

Evaluate $\lim_{x \to 0} (\sin x)^{\tan x}$

Solution

Putting $x = 0$ in the given expression, it reduces to the form -

$(\sin x)^{\tan x} = (\sin 0)^{\tan 0} = 0^{\infty}$

0^{∞} is indeterminate form.

Let $y = (\sin x)^{\tan x}$

$\log y = \log (\sin x)^{\tan x}$

$\log y = \tan x \log (\sin x)$

$$\lim_{x \to 0} \log y = \lim_{x \to 0} [\tan x \log \sin x]$$

$$= \lim_{x \to 0} \frac{\log \sin x}{\cot x} \quad \left[\frac{\infty}{\infty}\right]$$

By L. Hospital rule.

$$= \lim_{x \to 0} \frac{\frac{1}{\sin x} \cdot \cos x}{-\operatorname{cosec}^2 x}$$

$$= \lim_{x \to 0} \left[-\frac{\cot x}{\operatorname{cosec}^2 x}\right] \quad \left[\because \text{when } x \to 0 \ \frac{-\infty}{\infty}\right]$$

Again L. Hospital rule using

$$= \lim_{x \to 0} \left[\frac{+\operatorname{cosec}^2 x}{2 \operatorname{cosec} x \cdot (\operatorname{cosec} x \cot x)}\right]$$

$$= \lim_{x \to 0} \left[+\frac{1}{2} \frac{1}{\cot x}\right] = \lim_{x \to 0} \left(-\frac{1}{2} \tan x\right) = 0$$

$$\text{or} = \lim_{x \to 0} \left[\frac{1}{2 \operatorname{cosec}^2 x}\right] = \lim_{x \to 0} \left[\frac{1}{2} \sin^2 x\right] = 0$$

$\therefore \log y = 0$

$\log y = \log 1 \qquad [\because \log 1 = 0]$

$y = 1$

$\therefore$ Hence $\lim\limits_{x \to 0} (\cos x)^{\tan x} = 1$

Exp. Evaluate $\lim\limits_{x \to a} \left(2 - \frac{x}{a}\right)^{\tan \frac{\pi x}{2a}}$

Sol. putting $x = a$ in given expression then indeterminate form is 1^{∞}

Let $y = \left(2 - \frac{x}{a}\right)^{\tan \frac{\pi x}{2a}}$

$\log y = \tan \frac{\pi x}{2a} \log\left(2 - \frac{x}{a}\right)$

$$\lim_{x \to a} \log y = \lim_{x \to a} \frac{\log\left(2 - \frac{x}{a}\right)}{\cot \frac{\pi x}{2a}} = \frac{\log\left(2 - \frac{a}{a}\right)}{\cot \frac{\pi}{2}} = \frac{0}{0}$$

Using L.H Rule.

$$= \lim_{x \to a} \frac{\frac{1}{\left(2 - \frac{x}{a}\right)} \cdot \left(-\frac{1}{a}\right)}{-\operatorname{cosec}^2 \frac{\pi x}{2a} \cdot \frac{\pi}{2a}}$$

$$= \lim_{x \to a} \frac{2}{\left(2 - \frac{x}{a}\right) \cdot \operatorname{cosec}^2 \frac{\pi x}{2a} \cdot \pi}$$

$$= \frac{2}{1 (\operatorname{cosec} \frac{\pi}{2})^2 \cdot \pi} = \frac{2}{\pi}$$

$\lim\limits_{x \to a} \log y = \frac{2}{\pi} \Rightarrow \lim\limits_{x \to a} y = e^{2/\pi}$

$\lim\limits_{x \to a} y = e^{2/\pi}$