Viscosity

Viscosity is the property of a fluid by virtue of which an internal frictional forces comes into play when the fluid is in motion and opposes the relative motion of its different layers.

To understand it, let us stir water in a beaker with a glass rod. The water rotates in coaxial cylindrical layers, fig 1. Put some pieces of paper on the surface of rotating water. Their movements show that the speed of the inner most cylindrical layer is maximum. The speed of successive layers goes on decreasing as we approach the walls of the beaker. Thus there exists a relative motion amongst different layers of water. If we stop stirring, water continues rotating for some time and then stops. According to Newton's first law of motion, the liquid can stop only when an opposing force is acting on the layers which destroys their relative motion. Since no force is applied from outside, the opposing force must be arising from within the liquid. This opposing force is called Viscous force and this property of liquid is called viscosity.

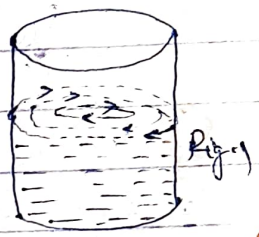
Fig. 1

Cause of viscosity: Viscosity is due to the inter-molecular forces which are effective when the different layers of the liquid are moving with different velocities. These forces are of Vander Waal type and vary inversely as the seventh power of intermolecular distance. Due to these forces, every fast moving liquid layer tends to accelerate the adjoining slow moving layer and every slow moving layer tends to retard the adjoining fast moving layer of liquid. As a result a backward dragging force called viscous drag comes into play which accounts for viscosity of liquid.

Coefficient of viscosity

Consider a liquid flowing steadily over a fixed horizontal surface. It flows in the form of layers. The layer immediately in contact with the surface moves with the minimum velocity due to force of adhesion between liquid and solid surface. The velocity of other layers goes on increasing as the distance from the fixed surface increases. Thus the topmost layer moves with maximum velocity Fig. 2.

Consider two adjoining layers P and Q. The layers Q moving faster than P, tries to accelerate P. The layer P, moving at a slower rate tries to retard Q. Thus two layers tend to destroy their relative motion as if there is a backward dragging force acting tangentially on the layers. Therefore to maintain the flow of liquid, an external force equal and opposite to backward dragging force must be applied.

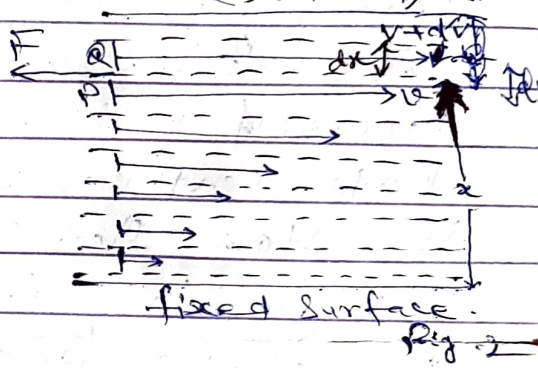

Fig. 2

Newton observed that a backward tangential force, F acting between two layers of liquid moving with relative velocity dv with respect to each other, separated by a distance dx is given by

(i) $F \propto A$ (A = Area of each layer)

(ii) $F \propto \frac{dv}{dx}$

($\frac{dv}{dx}$ = velocity gradient between the layers)

Thus $F \propto A \frac{dv}{dx}$

or, $F = -\eta A \frac{dv}{dx}$ — (1)

where η is a constant and is called the coefficient of viscosity of the liquid. The negative sign shows that the direction of F is opposite to the direction of velocity.

If $A = 1$, $dv/dx = 1$, then from (1)

$F = -\eta$ or, $\eta = -F$

Thus coefficient of viscosity of a liquid is defined as the tangential force required to maintain a unit velocity gradient between two layers of unit area.

Dimensional formula for η.

From (1), $\eta = \frac{f}{A\, dv/dx}$

$= \frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]} = [ML^{-1}T^{-1}]$

Units of coefficient of viscosity:

From (1), $\eta = \frac{f}{A(dv/dx)}$ — (2)

In cgs system, the unit of η is called poise

From (2), 1 poise $= \frac{1 \text{ dyne}}{1\ cm^2 \times (1\ cm\ s^{-1}/cm)}$

$= 1$ dyne cm^{-2} sec.

The S.I. unit of η is Pa s (Pascal-Second) or decapoise. From (2),

1 decapoise $= \frac{1 \text{ newton}}{1 m^2 \times (1 m s^{-1}/m)} = 1\ N s m^{-2}$

STOKE'S LAW

When a small spherical body moves through a viscous medium at rest, the layers of the body touching the medium are dragged along with it. But the layers of the medium away from the body are at rest. This causes a relative motion between different layers of medium. As a result of this, a backward dragging force comes into play, which opposes the motion of the body. This backward dragging force increases with the increase in velocity of the moving body.

Stoke found that the backward dragging force F acting on a small spherical body of radius r,

moving through a medium of coefficient of viscosity η, with velocity v is given by

$$F = 6\pi\eta r v \quad \text{——— (1)}$$

This is called Stokes law of viscosity. The relation (1) can be easily derived by the method of dimensions. Stoke observed that viscous drag (F) depends upon

(i) coefficient of viscosity (η) of the medium,
(ii) velocity (v) of the body, (iii) radius (r) of the spherical body.

Let $F = K \eta^a v^b r^c$ ——— (2)

where a, b and c are the dimensions of F in terms of η, v and r, and K is a dimensionless constant.

In dimensional form, the relation (2) can be written as

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^a [LT^{-1}]^b [L]^c$$
$$= [M^a L^{-a+b+c} T^{-a-b}]$$

Applying the principle of homogeneity of dimensions, we get

Powers of M, $a = 1$

Powers of L, $-a+b+c = 1$

or, $-1+b+c = 1$ or, $b+c = 2$ ——— (3)

Powers of T, $-a-b = -2$ or, $-1-b = -2$ or $b = 1$

from (3), $\therefore 1+c = 2$ or, $c = 1$

Substituting the values of a, b and c in (2), we get

$$F = K\eta r v$$

The value of K was found to be 6π, experimentally.

$$\boxed{F = 6\pi\eta r v}$$

Importance of Stokes law

1. This law is used in the determination of electronic charge with the help of Millikan's experiment.
2. This law accounts the formation of clouds.
3. This law accounts, why the speed of rain drop is less than that of a body falling freely with a constant velocity from the height of clouds.
4. This law helps a man coming down with the help of a parachute.