VECTOR CALCULUS

(1) The necessary and sufficient condition for a vector function $u(t)$ to have a constant magnitude is $\vec{u} \cdot \frac{d\vec{u}}{dt} = 0$

i.e. $\vec{u}$ and its derivative $\frac{d\vec{u}}{dt}$ are mutually perpendicular

Proof :-

Necessity

Let $|u(t)| =$ constant $= a$

Then $\vec{u} \cdot \vec{u} = a^2$

Differentiating w.r.t. t, we get

$$u \cdot \frac{d\vec{u}}{dt} = 0$$

$\therefore$ The condition is necessary.

Sufficiency

Conversely, suppose $\vec{u} \cdot \frac{d\vec{u}}{dt} = 0$

$$2\vec{u} \cdot \frac{d\vec{u}}{dt} = 0$$

Then

$$\therefore \vec{u} \cdot \frac{d\vec{u}}{dt} + \frac{d\vec{u}}{dt} \cdot \vec{u} = 0$$

$$\therefore \frac{d}{dt} + (\vec{u} \cdot \vec{u}) = 0$$

$$\therefore \frac{d}{dt} (|\vec{u}|^2) = 0$$

$\therefore |\vec{u}| =$ Constant i.e. $\vec{u}$ has a Constant magnitude