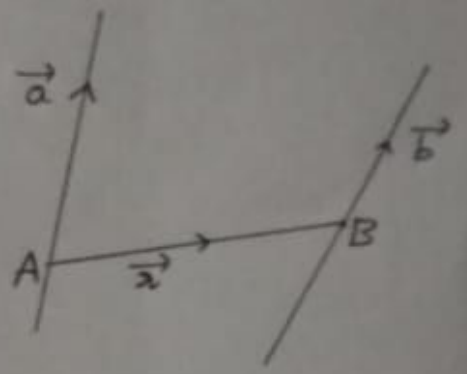
VECTOR ALGEBRA

Q⑥ If $\vec{a}$ and $\vec{b}$ are two vectors prove that a times the moment of $\vec{b}$ about the line of action of $\vec{a}$ is equal to b times the moment of $\vec{a}$ about the line of action of $\vec{b}$.

The moment of $\vec{b}$ about the line of action of $\vec{a}$ is

$$S_1 = [\hat{a}\ \vec{r}\ \vec{b}]$$



where

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{a}$$

and $\vec{r} = \overrightarrow{AB}$, A being any point on $\vec{a}$ and B being any point on $\vec{b}$.

$$\therefore aS_1 = a\{\hat{a} \cdot \vec{r} \times \vec{b}\}$$
$$= \{\vec{a} \cdot \vec{r} \times \vec{b}\}$$
$$= [\vec{a}\ \vec{r}\ \vec{b}] \quad \text{——— ①}$$

A being any point on $\hat{a}$ and B being any point on $\vec{b}$.

Similarly if S_2 be the moment of $\vec{a}$ about the line of action of $\vec{b}$, then $S_2 = [\hat{b}\ -\vec{r}\ \vec{a}] = -[\hat{b}\ \vec{r}\ \vec{a}]$

$$\therefore bS_2 = -\vec{b} \cdot \vec{r} \times \vec{a} = -[\vec{a}\ \vec{b}\ \vec{r}] = [\vec{a}\ \vec{r}\ \vec{b}]$$

——— ②

From ① and ② we get the desired result

$$aS_1 = bS_2$$