Paper 1, TDC Part-1
Chapter– 2, Complex Algebra and J operator

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Complex Algebra and J operator

Properties of j Operator

$j = 90^\circ$ counter-clock wise rotation $= \sqrt{-1}$

$j^2 = 180^\circ$ " – " " " $= -1$

$j^3 = 270^\circ$ " – " " " $= (\sqrt{-1})^3$

$j^4 = 360^\circ$ " – " " " $= (\sqrt{-1})^4 = (-1)^2 = +1$

$j^5 = 450^\circ$ " " " " $= (j^4) \cdot j = j = \sqrt{-1}$

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Also note that

$$\frac{1}{J} = \frac{J}{J^2} = \frac{J}{-1} = -J$$

So, $\frac{1}{J} = -J$

This letter 'J' used in the expression is a symbol of an operation.. Just as symbols '$\times$', '$+$', $\int$ etc. Symbol 'j' in a expression indicate counter-clockwise rotation of a vector through 90°.

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~~Ex~~ Complex Algebra :→

A complex quantity has both magnitude and direction and it is specified in vector form. To ~~describe~~ specify a vector quantity we need to 2 components → X – Component & Y – component. X – component indicates or shows horizontal component of vector and y – component specifies vertical component.

Consider a vector given as

$$\overline{OE_1} = a_1 + j b_1$$

This vector $\overline{OE_1}$ has 2 - component one is horizontal component which is a_1 unit

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and j indicates that b_1 is ⊥r to component a_1.

The vector written in this way is said to be written in "Complex form".

In Mathematics, a_1 is known as real component and b_1 as $\boxed{\text{imaginary component}}$ but in electrical engineering, these are known as in phase (or active) and $\boxed{\text{quadrature (or reactive)}}$ components respectively

The numerical value of vector $OE_1 = \sqrt{a_1^2 + b_1^2}$.

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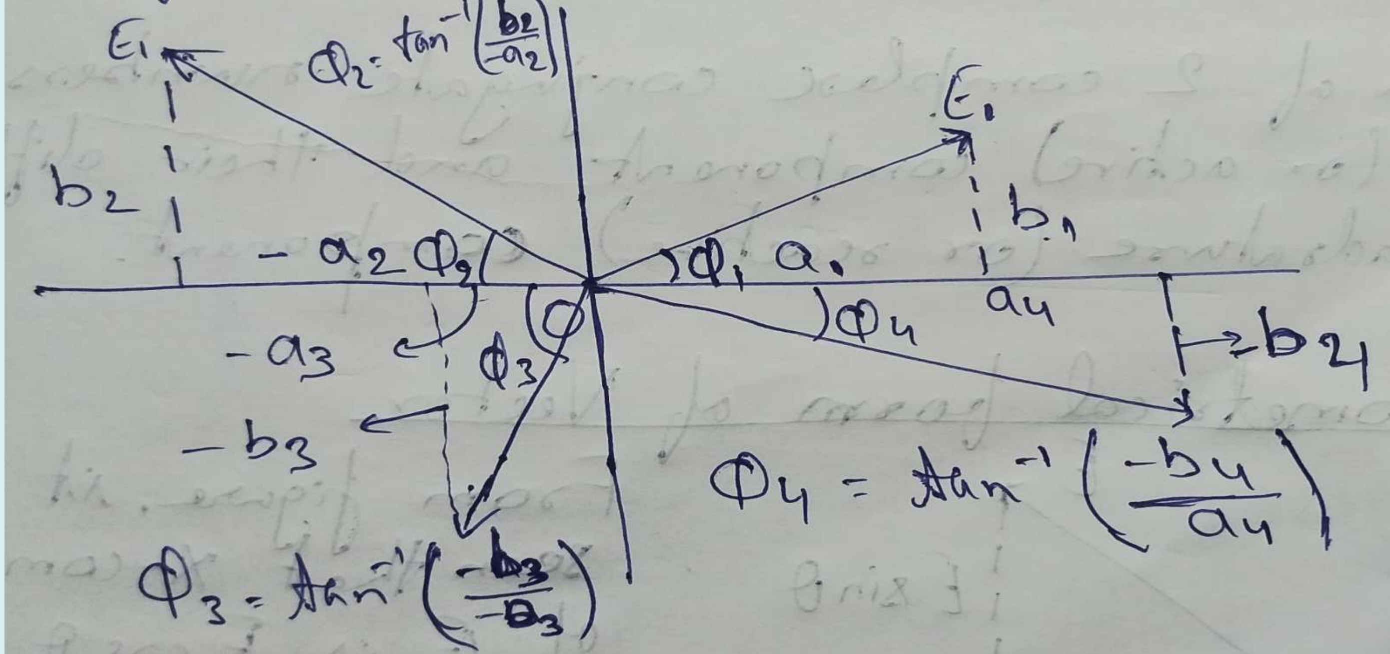
Its angle with X-axis is given by $\Phi_1 = \tan^{-1}(b_1/a_1)$

$\overline{OE_1} = a_1 + jb_1$ is ~~kn~~ cartesian for representation of ~~a~~. a vector.



The other vectors have also been ~~repres~~ represented in above diagram.

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These vectors can be similarly expressed as

$\overline{OE_2} = \overline{E}_2 = -a_2 + jb_2$ and $\Phi_2 = \tan^{-1}\left(\frac{-b_2}{a_2}\right)$

$\overline{OE_3} = \overline{E}_3 = -a_3 - jb_3$ & $\Phi_3 = \tan^{-1}\left(\frac{+b_3}{+a_3}\right)$

$\overline{OE_4} = \overline{E}_4 = +a_4 - jb_4$ & $\Phi_4 = \tan^{-1}\left(\frac{-b_4}{a_4}\right)$

Conjugate Complex Number :-

Two complex numbers are said to be conjugate if they differ only in the algebraic sign of their quadrature components. For example

$\overline{OE_1}\ (\overline{E}_1)$ & $\overline{OE_4}\ (\overline{E}_4)$ are complex conjugate

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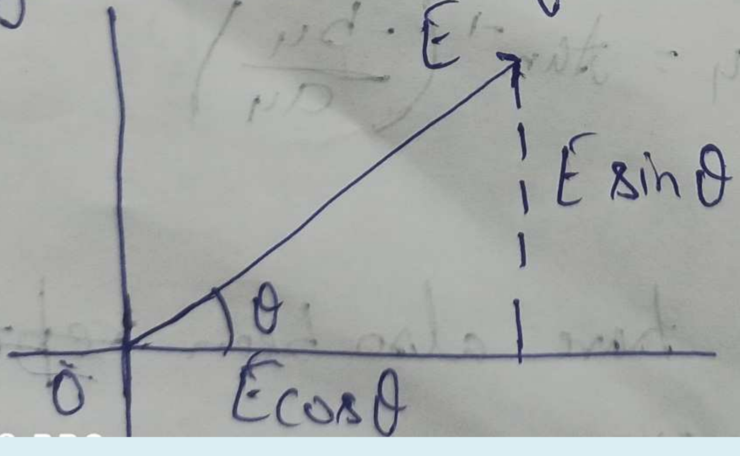
llarly
$\overline{OE_2}\,(\bar{E}_2)$ & $\overline{OE_3}\,(\bar{E}_3)$ are complex conjugate

The sum of 2 complex conjugate numbers gives in-phase (or active) component and their difference gives quadrature (or reactive) component.

Trigonometrical form of Vector



From figure, it is seen that X-component of E is E cos θ & Y-component is E sin θ

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~~Two complex~~ So we can express the vector $\overline{OE} = \bar{E}$ as

$$\bar{E} = E\cos\theta + jE\sin\theta = E(\cos\theta + j\sin\theta)$$

This is equivalent to the rectangular form

$\bar{E} = a + jb$ where,

$a = E\cos\theta$ &

$b = E\sin\theta$

<u>Exponential form of vector :→</u>

A vector $\overline{OE} = \bar{E} = E(\cos\theta + j\sin\theta)$

$= Ee^{j\theta}$

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also if $\overline{OP} = \bar{P} = E(\cos\theta - j\sin\theta)$

$= Ee^{-j\theta}$

as we have $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

||arly if $\overline{OQ} = \bar{Q} = -E\cos\theta + Ej\sin\theta$

$\bar{Q} = -E(\cos\theta - j\sin\theta)$

$\bar{Q} = -Ee^{-j\theta}$

||arly is $\overline{OR} = \bar{R} = -E\cos\theta - jE\sin\theta$

$= -E(\cos\theta + j\sin\theta)$

$= -Ee^{j\theta}$

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For any query contact- 9771474020

Thank You