Paper 1, TDC Part-1
Chapter– 2, Complex Quantity and J operator

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Complex Quantity and J Operator

Unit 2 → Complex Number & J-Operato

Alternating voltage or current is phasor quantity, having both magnitude and phase. By phase of an alternating voltage or current we mean that the fraction of the time period of that alternating voltage or current which has elapsed since the voltage has applied or current last passed, through the zero position of reference. for example

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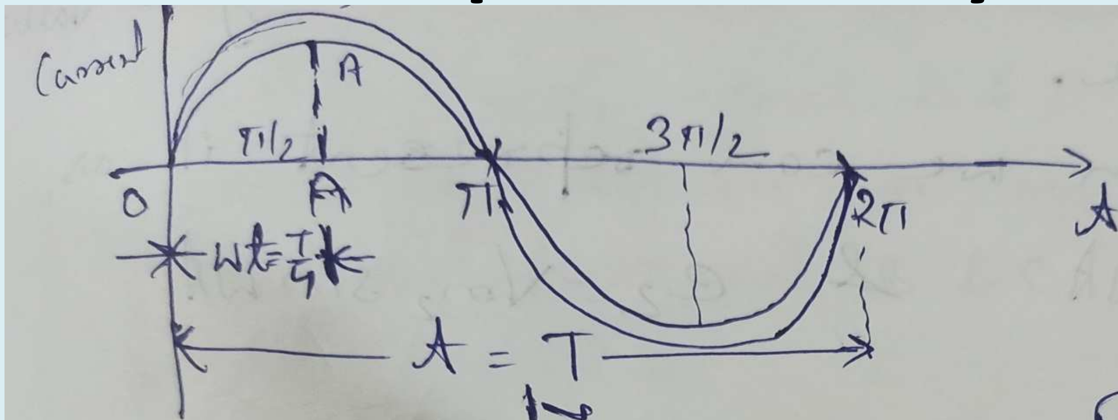
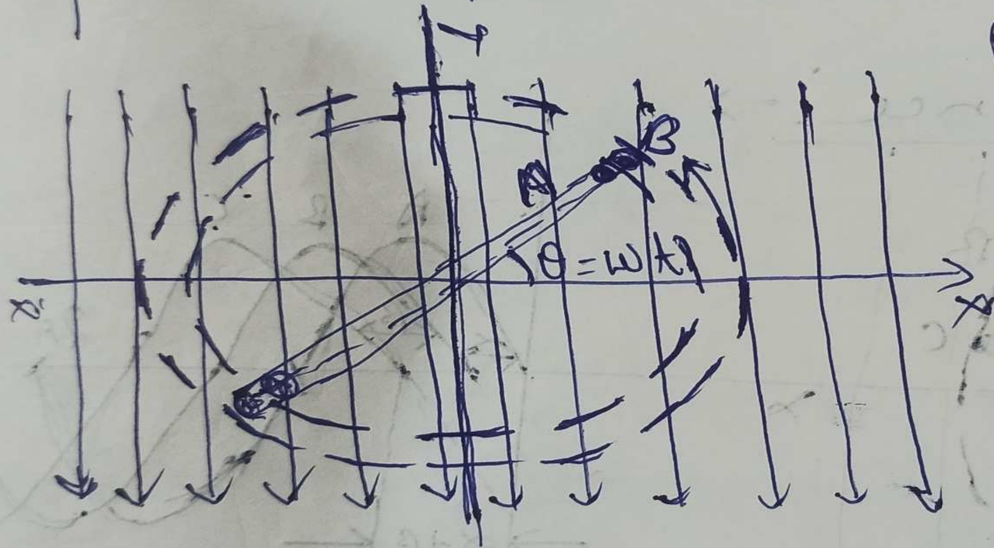


These time period of this current is $T = 2\pi$



For example, the phase of current at point A is $\frac{T}{4}$ second, or in terms of angle it is $\pi/2$ radians therefor it is called phase angle.

In electrical network we are more concerned with relative phases or phase differences between different alternating quantities.

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Here, consider two single-turn coils of different sizes arranged radially in the same plane and rotating with the same angular velocity in a common magnetic field of uniform intensity. So the induced e.m.fs in both coils will be of the same frequency and of sinusoidal shape, however the values of instantaneous e.m.fs induced would be different. Such alternating voltages (or current) are in phase with each other as they reach their maximum and zero values at same time.

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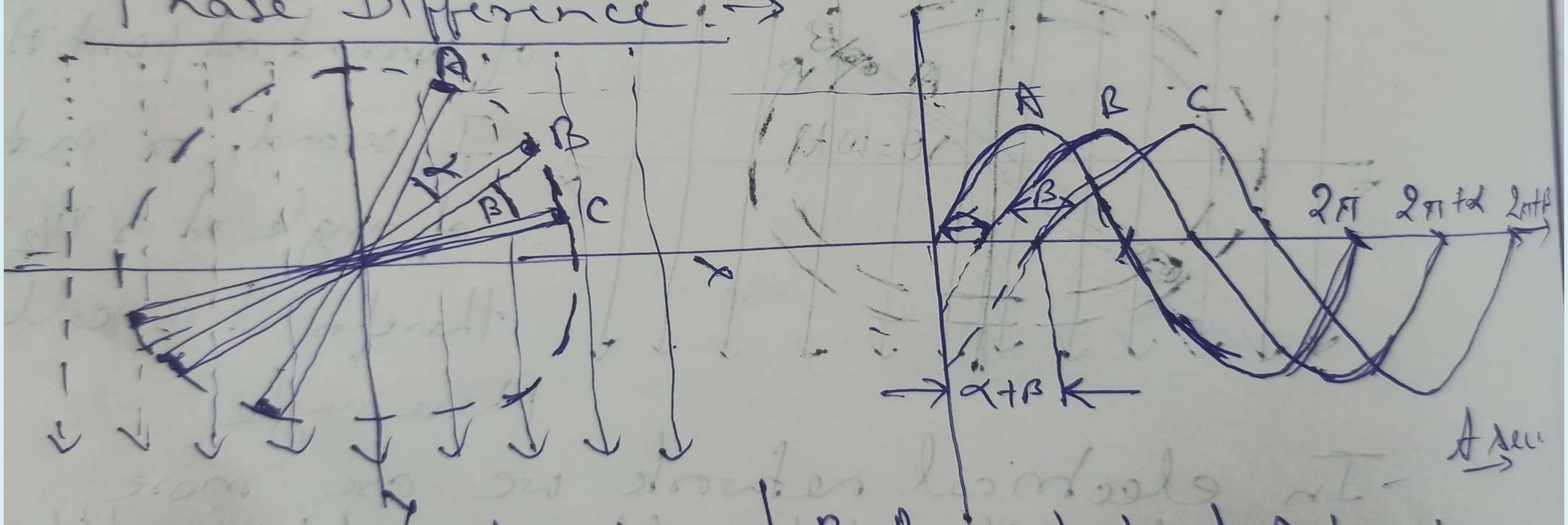
In eqn. form we can represent it as,

$e_1 = V_{m1} \sin \omega t$ & $e_2 = V_{m2} \sin \omega t$

Phase Difference :→



A leads signal B by α

B lags behind A by α
C " " B " β
C " " A " α+β

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Consider 3 similar coils displaced from each other by angles α and β and rotating uniform magnetic field with the same angular velocity.

In this case, the ~~value of~~ e.m.fs induced in the coils are of the same value. but their phases are different. The e.m.fs max$^{\underline{m}}$. or zero values are reached at different time so the phase angle of these signals are different.

Here signal A reaches it's maximum value first then signal B and followed by signal C. So the signal A leads signal B & C and signal B ~~and~~

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~~leads~~ signal ~~C~~. We can say that signal C lags signal behind signal B & C and signal B lags behind signal A.

A leading alternating quantity is one which ~~leads~~ reaches its maximum or zero value earlier as compared to the other quantity.

Illy, a lagging alternating quantity is one which reaches its maxm. or zero value later than the other quantity.

Based on this we can write expressions for

$e_A = E_m \sin \omega t$, $e_B = E_m \sin(\omega t - \beta)$

$e_C = E_m (\sin \omega t - \beta - \alpha)$

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In this fig A signal B leads A by an angle Φ.

Hence we can write these equations as,

$$e_A = E_m \sin \omega t$$

$$e_B = E_m \sin(\omega t + \Phi)$$

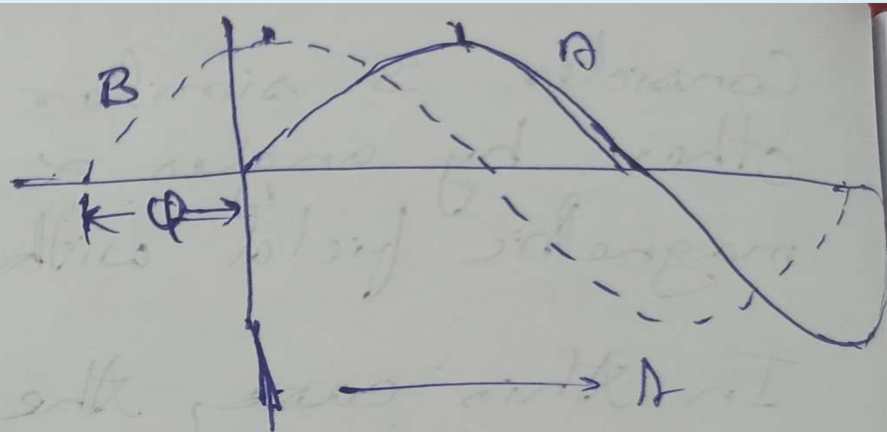

Figure A

So '+' sign in phase difference denotes lead whereas '−' sign denotes lag.

When alternating voltage i.e. phasor (complex) quantity is applied to a circuit, the resulting current is also a complex quantity. The property circuit obtained ... ratio of voltage ...

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as the ratio of voltage to current (i.e. impedance)
is also then a complex quantity. Thus the
analysis of ac circuits involves complex quantities.
To express complex quantities operator 'J' is used

$$j = \sqrt{-1}.$$

Any quantity multiplied by j means that the
quantity is rotated through 90° in the counter
clockwise direction.

i.e. when vector OA is operated with j, we
obtained the new vector OB, which is
displaced by 90° in counter clockwise

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direction from OA.

$$\overline{OB} = j\,\overline{OA}$$

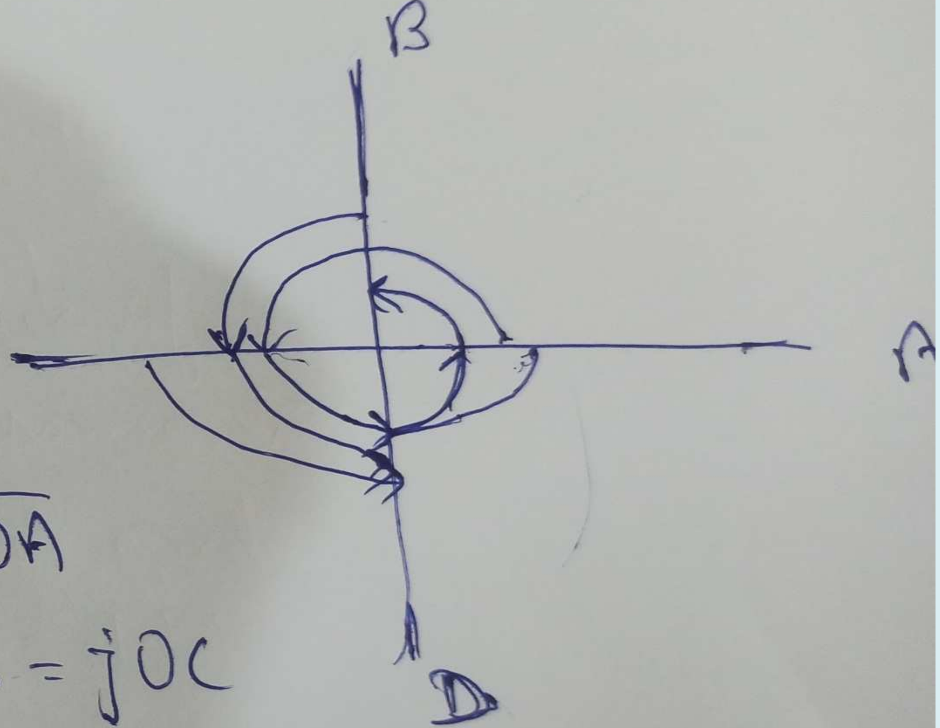
<ins>Similarly</ins>

Operation of j twic oA

$\overline{OA}$ gives $\overline{OC} = j^2\,\overline{OA} = j\,\overline{OB}$

Operation of j thrice on $\overline{OA}$

gives $\overline{OD} = j^3\,\overline{OA} = j^2\,\overline{OB} = j\,OC$



Operation of j four times on $\overline{OA}$ gives back $\overline{OA}$.

i.e. $\overrightarrow{OA} = j^4\,\overline{OA} = j^3\,OB = j^2\,OC = j\,OD$

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Operation of j four times on $\overline{OA}$ gives back $\overline{OA}$.

i.e. $\overrightarrow{OA} = j^4\ \overline{OA} = j^3\ OB = j^2\ OC = j\ OD$

Hence it is seen that successive operation of the operator j to the vector O produce successive 90° steps of rotation of vector in the counter clockwise direction. However the magnitude of the vector is not altered in any way.

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For any query contact- 9771474020

Thank You