Paper 1, TDC Part-1
Chapter– 2, Complex Algebra and J operator

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Complex Algebra and J operator

Multiplication and Division of Vector Quantities

Multiplication and division of vectors becomes very simple and easy if they are represented in the polar or exponential form.

Multiplication in Rectangular Form

Let 2 quantities to be multiplied are

$\bar{A}_1 = a_1 + jb_1$ & $\bar{A}_2 = a_2 + jb_2$

then $\bar{A}_1 \times \bar{A}_2 = (a_1 + jb_1)(a_2 + jb_2)$

$$\bar{A} = a_1 a_2 + j a_2 b_1 + j b_2 a_1 + j^2 b_1 b_2$$

$$\bar{A} = a_1 a_2 + j(a_1 b_2 + a_2 b_1) + (-1) b_1 b_2 \quad [j^2 = -1]$$

$$\bar{A} = (a_1 a_2 - b_1 b_2) + j(a_1 b_2 + a_2 b_1)$$

Complex Algebra and J operator

Magnitude of $\bar{A} = \sqrt{(a_1a_2 - b_1b_2)^2 + (a_1b_2 + a_2b_1)^2}$

the angle suspended by $\bar{A}$ with x-axis is →

$$\theta = \tan^{-1}\left(\frac{a_1b_2 + b_1a_2}{a_1a_2 - b_1b_2}\right)$$

(ii) Multiplication in Polar Form :→

Consider 2 complex number again to be multiplied are, $\bar{A}_1 = a_1 + jb_1$ & $\bar{A}_2 = a_2 + jb_2$

we can convert the given number $\bar{A}_1$ & $\bar{A}_2$ into it's polar form.

Magnitude of $A_1 = \sqrt{a_1^2 + b_1^2}$ & it's angle with x-axis θ_1

$$\theta_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right)$$

Magnitude of $A_2 = \sqrt{a_2^2 + b_2^2}$ & it's angle with y-axis $= \theta_2$

$$\theta_2 = \tan^{-1}\left(\frac{b_2}{a_2}\right)$$

Complex Algebra and J operator

So, $\bar{A}_1 = A_1 \angle\theta_1$ & $\bar{A}_2 = A_2 \angle\theta_2$

Where A_1 & A_2 are magnitude of $\bar{A}_1$ & $\bar{A}_2$ and θ_1 & θ_2 are angle substended by $\bar{A}_1$ & $\bar{A}_2$ with x-axis.

$$\bar{A}_1 \cdot \bar{A}_2 = A_1 \angle\theta_1 \cdot A_2 \angle\theta_2$$

$$= A_1 A_2 \angle\theta_1 + \theta_2$$

(ii) Multiplication in exponential form :-

The process of multiplication in exponential form is same as in polar form. We should first express the given ~~number~~ quantities in exponential form by finding magnitude & angle substended with x-axis, then multiply the given quantities

Complex Algebra and J operator

exponential form by finding magnitude & angle subtended with x-axis, then multiply the given quantities.

$\bar{A}_1 = a_1 + j b_1$

in exponential form $\bar{A}_1 = A_1 e^{j\theta_1}$

where $A_1 = \sqrt{a_1^2 + b_1^2}$ & $\theta_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right)$

$\bar{A}_2 = a_2 + j b_2$

in exponential form $\bar{A}_2 = A_2 e^{j\theta_2}$

where $A_2 = \sqrt{a_2^2 + b_2^2}$ & $\theta_2 = \tan^{-1}\left(\frac{b_2}{a_2}\right)$

Now $\bar{A} = \bar{A}_1 \cdot \bar{A}_2 = A_1 e^{j\theta_1} \cdot A_2 e^{j\theta_2}$

$\bar{A} = \bar{A}_1 \bar{A}_2 = A_1 A_2 e^{j(\theta_1 + \theta_2)}$

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Division of Complex Quantities :→

(i) Rectangular form :→

$$\frac{\bar{A}_1}{\bar{A}_2} = \frac{a_1 + jb_2}{a_2 + jb_2} = \frac{(a_1 + jb_1)(a_2 - jb_2)}{(a_2 + jb_2)(a_2 - jb_2)}$$

$$= \frac{(a_1a_2 + b_1b_2) + j(a_2b_1 - a_1b_2)}{(a_2^2 - j^2b_2^2)}$$

$$= \frac{(a_1a_2 + b_1b_2)}{(a_2^2 + b_2^2)} + j\,\frac{(a_2b_1 - a_1b_2)}{(a_2^2 + b_2^2)}$$

(ii) Polar Form :→

The complex quantity expressed in polar form can be divided as below :→

$$\frac{\bar{A}_1}{\bar{A}_2} = \frac{A_1 \angle \theta_1}{A_2 \angle \theta_2} = \frac{A_1}{A_2} \angle \theta_1 - \theta_2$$

Complex Algebra and J operator

(iii) Exponential Form :→

Like in polar form division in exponential form is also same as expressed below:→

$$\frac{\bar{A}_1}{\bar{A}_2} = \frac{A_1 e^{j\theta_1}}{A_2 e^{j\theta_2}} = \frac{A_1}{A_2} e^{j(\theta_1-\theta_2)}$$

Power and Roots of Complex Quantity

(a) Powers :→ Like multiplication power & roots are easy to find out in polar & exponential form.

· The 'N' power of any complex quantity expressed

Complex Algebra and J operator

in polar form can be given

by $(\bar{A}_1)^N = (A_1 \angle \theta_1)^N = A^N \angle N\theta_1$

Let $\bar{A}_1 = 5 \angle 45^\circ$

$(\bar{A}_1)^4 = 5^4 \angle 4 \times 45^\circ = 625 \angle 180^\circ$

$(\bar{A}_2)^3 = (10 \angle -20^\circ)^3 = 10^3 \angle -20 \times 3 =$

$10 \angle -20$ $\qquad = 1000 \angle -60^\circ$

$\bar{A}^N \bar{B}^M$ can be found in similar manner

Let $\bar{A} = A \angle \theta_1$ & $\bar{B} = B \angle \theta_2$

Complex Algebra and J operator

Then $(\bar{A}^N \cdot \bar{B}^M) = (A \angle\theta_1)^N (B \angle\theta_2)^M$

$= A^N \angle N\theta_1 \cdot B^M \angle M\theta_2$

$= A^N \cdot B^M \angle(N\theta_1 + M\theta_2)$

(ii) Roots

$(\bar{A})^{1/N} = (A \angle\theta_1)^{1/N_1} = A^{1/N_1} \angle\theta_1/N_1$

Let $\bar{A} = (256 \angle 200^\circ)^{1/4}$

$= \sqrt[4]{256} \cdot \angle 200^\circ/4$

$= 4 \angle 50^\circ$

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Example 1) Given two current $i_1 = 10\sin(\omega t + \pi/4)$ & $i_2 = 5\cos(\omega t - \pi/2)$. Find the r.m.s value of $i_1 + i_2$ using the complex number representation.

Soln:

$$i_2 = 5\cos(\omega t - \pi/2)$$

$$= 5\sin\left[90^\circ + (\omega t - \pi/2)\right] \qquad [\cos\theta = \sin(90^\circ + \theta)]$$

$$i_2 = 5\sin(\omega t) \qquad [90^\circ - \pi/2 = 0]$$

So current i_2 is in phase with reference quantity. Max^{m} value of $i_2 = 5$ A

Complex Algebra and J operator

$$I_{m1} = 10(\cos 45^\circ + j\sin 45^\circ) = 7.07 + j7.07$$

Ilarly $I_{m2} = 5(\cos 0^\circ + j\sin 0^\circ) = 5 + j0$

Resultant current I is $i_1 + i_2$ given by,

$$\bar{I} = 7.07 + j7.07 + 5 + j0$$

$$= (12.07 + j7.07)$$

$$I = \sqrt{(12.07)^2 + (7.07)^2} = \sqrt{145.6849 + 49.985}$$

$$= \sqrt{195.6698} \simeq 14$$

R.M.S value of resultant current is $\Rightarrow \dfrac{14}{\sqrt{2}}$

$$= \underline{\underline{10A}}$$

Complex Algebra and J operator

12.11. The 120° Operator

In three-phase work where voltage vectors are displaced from one another by 120°, it is convenient to employ an operator which rotates a vector through 120° forward or backwards without changing its length. This operator is 'a'. Any operator which is multiplied by 'a' remains unchanged in magnitude but is rotated by 120° in the counter-clockwise (ccw) direction.

$$\therefore \quad \alpha = 1 \angle 120^\circ$$

This, when expressed in the cartesian form, becomes

$$a = \cos 120^\circ + j \sin 120^\circ = -0.5 + j\,0.866$$

Similarly, $a^2 = 1 \angle 120^\circ \times 1 \angle 120^\circ = 1 \angle 240^\circ = \cos 240^\circ + j \sin 240^\circ = -0.5 - j\,0.866$

Hence, operator 'a^2' will rotate the vector in ccw by 240°. This is the same as rotating the vector in *clockwise* direction by 120°.

$$\therefore \quad a^2 = 1 \angle -120^\circ. \text{ Similarly, } a^3 = 1 \angle 360^\circ = 1^*$$

As shown in Fig. 12.11, the 3-phase voltage vectors with standard phase sequence may be represented as E, a^2E and aE or as E, $E(-0.5 - j\,0.866)$ and $E(-0.5 + j\,0.866)$

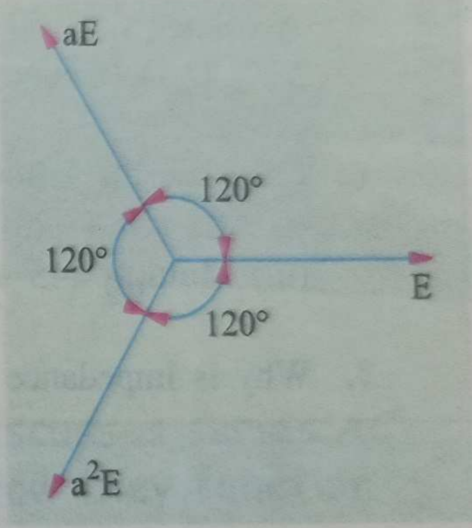

Fig. 12.11

It is easy to prove that

(*i*) $a^2 + a = -1$ (*ii*) $a^2 + a + 1 = 0$ (*iii*) $a^3 + a^2 + a = 0$

Complex Algebra and J operator

For any query contact- 9771474020

Thank You