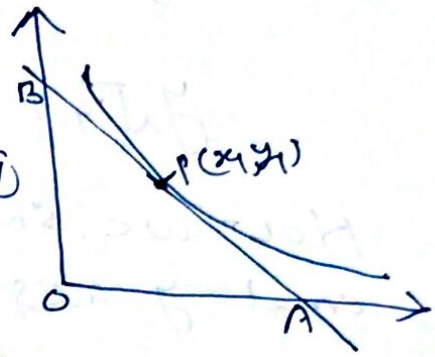
Exp. Show that the sum of the intercepts of the tangent to $\sqrt{x}+\sqrt{y}=\sqrt{a}$ upon the Co-ordinate axes is constant.

Sol: Let $P(x_1, y_1)$ be any point on the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$

$\because$ Then the Equation of the tangent at $P(x_1, y_1)$ will be



$$y-y_1=\left(\frac{dy}{dx}\right)_{\substack{x=x_1\\y=y_1}}(x-x_1) \quad -①$$

given curve

$$\sqrt{x}+\sqrt{y}=\sqrt{a}$$

$$\frac{1}{2}\cdot\frac{1}{\sqrt{x}}+\frac{1}{2}\frac{1}{\sqrt{y}}\cdot\frac{dy}{dx}=0$$

$$\frac{1}{\sqrt{y}}\frac{dy}{dx}=-\frac{1}{\sqrt{x}}$$

$$\frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)_{\substack{x=x_1\\y=y_1}}=-\frac{\sqrt{y_1}}{\sqrt{x_1}}$$

Putting in ① we get

$$y-y_1=-\frac{\sqrt{y_1}}{\sqrt{x_1}}(x-x_1)$$

$$y\cdot\sqrt{x_1}-y_1\cdot\sqrt{x_1}=-\sqrt{y_1}\cdot x+\sqrt{y_1}\cdot x_1$$

$$y\cdot\sqrt{x_1}+x\cdot\sqrt{y_1}=y_1\cdot\sqrt{x_1}+\sqrt{y_1}\cdot x_1$$