Similarity of Matrics :- Let A and B are two matrics. Then the matrix B is said to be similar to A, If there exists an invertible matrix P such that $B = P^{-1}AP$.

Theorem :- Similarity of matrics is an equivalence relation.

Proof :- Equivalence relation is reflexive, symmetric and transtive.

(i) **Reflexive :-** Let A be any square matrix of order $n \times n$ and I_n be the unit (or Identity) matrix of order $n \times n$, Then I_n is invertible as I_n^{-1} and $I_n^{-1} = I_n$, So $A = I_n^{-1} A I_n$, This is similar to A.

(ii) **Symmetric :-** Let A and B be any two square matrices of order $n \times n$ and if A is similar to B, then there exists an invertible matrix P, such that

$$A = P^{-1} B P$$
$$\Rightarrow B = PAP^{-1} \Rightarrow (P^{-1})^{-1} A P^{-1}$$

Since P^{-1} is invertible. Thus B is similar to A.

(iii) **Transitive :-** Let A, B & C be three square matrces of order $n \times n$ and If A is similar to B and B is similar to C there exist two invertible matrices P and Q such that $A = P^{-1}BP$ and $B = Q^{-1}CQ$

Now $A = P^{-1}BP \Rightarrow A = P^{-1}(Q^{-1}CQ)P$

$$\Rightarrow A = P^{-1}Q^{-1}CQP$$
$$\Rightarrow A = (QP)^{-1} CQP$$

Since QP is invertible, that A is similar to C.

Hence, the similarity of matrics is reflexive, symmetric and transtive, consequently similarity of matrices is an equivalance relation.

Similarity of Linear Transformation :-

Let V be an n-dimensional vector space over a field F and $A(V)$ be the set of all linear transformations from V to V. Then two linear transformations $S, T \in A(V)$ are said to be <u>similar</u> if there exists an invertible L.T. $C \in A(V)$ such that

$$T = CSC^{-1}$$

The relation on $A(V)$ defined by similarity is an equivalance relation, thus $A(V)$ decomposes into equivalance classes, each class is called similarity class.

Equivalance relation means:-

(i) Reflexive i.e $A = I_n^{-1} A I_n$

(ii) Symmetric i.e $A = P^{-1}BP$ or $B = PAP^{-1}$

(iii) Transitive i.e $A = P^{-1}BP$ and $B = Q^{-1}CQ$ $\Rightarrow A = C$.

The existence of linear transformations in each similarity class whose matrix representation in some basis of V is of special form, such matrixes are known as canonical forms. There are many canonical forms such as-

(i) Triangular form $\rightarrow$ $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$

(ii) Jordan form $\rightarrow$ $A = \begin{bmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \end{bmatrix}$

(iii) Rational Canonical form. $A = \begin{bmatrix} C_1 & & & \\ & C_2 & & \\ & & \ddots & \\ & & & C_n \end{bmatrix}$

[The matrix $A = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ is known as normal or canonical form.]