SPACE QUANTISATION

Classically, the orbits of atomic electrons can orient in all possible directions in space but Quantum theory allows only certain discrete orientations in space out of infinite possible orientations. This is known as space quantisation.

To specify the quantisation of orbits in space, a reference direction is required. Generally, a magnetic field is imagined to be along the z axis and the allowed orientations of this orbit are determined relative to that magnetic field direction. The space quantization of an electron orbit is specified by the projection of its total angular momentum on to reference direction, such projections being themselves quantised. The spin motion of the electrons are follows space quantisation.

(a) Orientation of the orbit →

When the electron orbit of an atom is placed in a strong magnetic field B, its angular momentum vector L interacts with the magnetic field through the magnetic moment vector μ. L can take only such orientations in space for which the projection of L_z along $\vec{B}$ will be integral multiple of $\hbar$ (see in next page) such that, $L_z = m_l \hbar$ where, m_l is called the orbital magnetic quantum number. It's possible values are

$$m_l = l, l-1, l-2, \ldots 0, \ldots, -l.$$

i.e, there are $(2l+1)$ value of m_l (see fig in next page)

The angle between l_z and l is given by,

$$\cos\theta = \frac{L_z}{L} = \frac{m_e\hbar}{l\hbar} = \frac{m_e}{l} \quad \text{--- (1)}$$

Since, according to wave mechanics l is replaced by $\sqrt{l(l+1)}$

$$\therefore \cos\theta = \frac{m_e}{\sqrt{l(l+1)}} \quad \text{--- (2)}$$

The torque due to the magnetic field causes the vector L to precess about the direction of B maintaining the same orientation θ as shown in the second fig. on this page.

The magnetic energy of the rotating electron will be,

$$\Delta E = \mu B\cos\theta \quad \text{--- (3)}$$

But, $\cos\theta = m_e/l$ and $\mu = \frac{e\hbar}{2mc}\cdot l$.

$$\Delta E = \frac{e\hbar}{2mc}\, lB\, m_e/l$$

$$= e\hbar B m_e/2mc \quad \text{--- (4)}$$

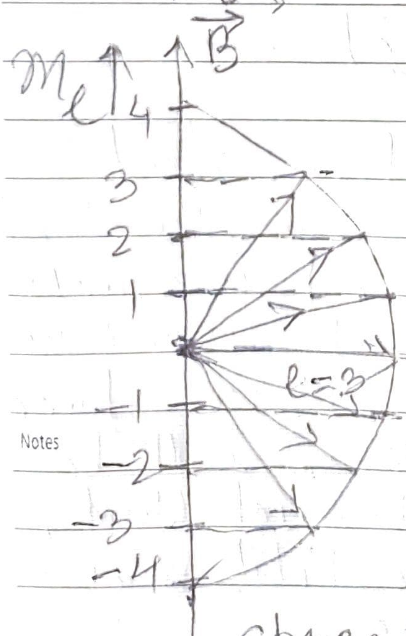
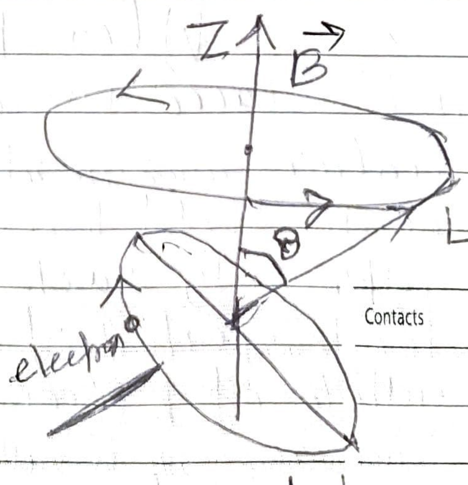


fig (1st)

space quantization

fig (Ist)



L precesses about Z direction

fig (2nd).

But,

$$\Delta E = m_l h \nu_L \text{ or}$$

or, $$\nu_L = \frac{eB}{4\pi mc} \quad \text{—— (5)}$$

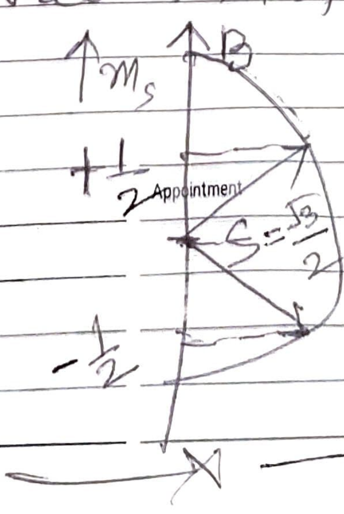
where, $\nu_L \Rightarrow$ Larmor precessional frequency.

(b) Spin of Electron →

The spin angular momentum vector S of the electron can assume only two possible orientations relative to the magnetic field $\vec{B}$ (fig. below). The projection of S along $\vec{B}$ is denoted by m_s, and is called magnetic spin quantum no. m_s can have only two values $+\frac{1}{2}$ and $-\frac{1}{2}$ corresponding to spin-up and spin-down orientations. Thus, the projection of the z component of S, i.e, S_z is given by

$$S_z = m_s \hbar \quad \text{—— (6)}$$

S_z can take $(2s+1) = (2 \times \frac{1}{2} + 1) = 2$ discrete values in space with respect to the magnetic field $\vec{B}$.



(Space quantisation of spin angular momentum for a single electron).