In the processes, we have decided for interpolation value which one of these formula gives the most accurate value. (1)

(i) If interpolation is desired near the beginning or end of a table, there is no alternative to Newton's forward and backward difference formulae.

(ii) For interpolation near the middle of a table, Stirling's formula gives the most accurate result for p lies $-\frac{1}{4} \le p \le \frac{1}{4}$ and Bessel's formula is most efficient near $p = \frac{1}{2}$, say $\frac{1}{4} \le p \le \frac{3}{4}$.

Exp. Find Solution using Gauss forward formula for given table

x	310	320	330	340	350	360
$f(x)$	2.4914	2.5052	2.5185	2.5315	2.5441	2.5563

at $x = 337.5$

Solution As $h = x_1 - x_0 \Rightarrow 320 - 310 = 10$

Taking $x_0 = 330 \Rightarrow p = \frac{x - x_0}{h} = \frac{337.5 - 330}{10} = 0.75$

Now the central difference Table

x	$p=\frac{x-x_0}{h}$	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
310	−2	2.4914					
			0.0138				
320	−1	2.5052		−0.0005			
			0.0133		0.0002		
330	0	2.5185 y_0		−0.0003 $\Delta^2 y_{-1}$		−0.0003 $\Delta^4 y_{-2}$	
			0.0130 Δy_0		−0.0001 $\Delta^3 y_{-1}$		0.0004 $\Delta^5 y_{-2}$
340	1	2.5315		−0.0004		0.0001	
			0.0126		0.00		
350	2	2.5441		−0.0004			
			0.0122				
360	3	2.5563					

Using Gauss's forward interpolation at $x = 337.5$ and $p = 0.75$

$$y(p) = y_0 + p\Delta y_0 + \frac{p(p-1)}{\lfloor 2}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{\lfloor 3}\Delta^3 y_{-1} + \frac{(p+1)p(p-1)(p-2)}{\lfloor 4}\Delta^4 y_{-2} + \frac{(p+2)(p+1)p(p-1)(p+2)}{\lfloor 5}\times\Delta^5 y_{-2}$$

Putting $p = 0.75$ and $y_0, \Delta y_0, \Delta^2 y_{-1}, \Delta^3 y_{-1}; \sim$ we obtain -

$$y(0.75) = 2.5185 + (0.75)(0.013) + \frac{(0.75)(0.75-1)(-0.0003)}{2} + \frac{1}{6}(0.75+1)(0.75)(0.75-1)(-0.0001) + \frac{(0.75+1)(0.75)(0.75-1)(0.75-2)(-0.0003)}{24} + \frac{(0.75+2)(0.75+1)(0.75)(0.75-1)(0.75-2)(0.0004)}{120}$$

$$y(0.75) = 2.5185 + 0.00975 + 0.000028125 + 0.0000054687 + 0.00000037598 - 0.00000067727$$

$y(0.75) = 2.52828$ is solution of $y(337.5)$

<u>Exp</u>: Find solution using Gauss's backward formula for table

x	1940	1950	1960	1970	1980	1990
y	17	20	27	32	36	38

at $x = 1976$

<u>Solution</u> From Table.

$h = 1950 - 1940 = 10$

Taking $x_0 = 1970$ then $p = \frac{1976-1970}{10}$

$p = 0.6$ because x is near to 1970 central value.

Now the central table

x	$p = \frac{x-x_0}{h}$	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1940	-3	17					
			3				
1950	-2	20		+4			
			7		-6		
1960	-1	27		-2		7	
			$5 \; \Delta y_{-1}$		$1 \; \Delta^3 y_{-2}$		$-9 \; \Delta^5 y_{-3}$
1970	$0 \to$	$32 \; y_0$		$+1 \; \Delta^2 y_{-1}$		$-2 \; \Delta^4 y_{-2}$	
			4		-1		
1980	1	36		-2			
			2				
1990	2	38					

using Gauss's Backward formula for Interpolation at $x = 1976$ where $p = 0.6$ then

$$y_{(p)} = y_0 + p\Delta y_{-1} + \frac{(p+1)p}{\underline{2}}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{\underline{3}}\Delta^3 y_{-2} + \frac{(p+1)p(p-1)(p-2)}{\underline{4}}\Delta^4 y_{-2} + \frac{(p+2)(p+1)p(p-1)(p-2)}{\underline{5}}$$

$$\times \Delta^5 y_{-3}.$$

putting the value of p and differences operators value in Eqn then from Gauss's Backward formula.

$$y(p) = y_0 + p\,\Delta y_{-1} + \frac{(p+1)p}{\angle 2}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{\angle 3}\Delta^3 y_{-2}$$

$$+ \frac{(p+1)p(p-1)(p-2)}{\angle 4}\Delta^4 y_{-2} + \frac{(p+2)(p+1)p(p-1)(p-2)}{\angle 5}\Delta^5 y_{-3}$$

then.

$$y(0.6) = 32 + (0.6)(5) + \frac{(0.6+1)(0.6)}{2}(-1)$$

$$+ \frac{(0.6+1)(0.6)(.6-1)}{6}(1) + \frac{(0.6+1)(0.6)(.6-1)(.6-2)}{24}(-2)$$

$$+ \frac{(.6+2)(.6+1)(.6)(.6-1)(.6-2)}{120}(-9)$$

$$y(0.6) = 32 + 3.0 - 0.48 - 0.064 + 0.0832 - 0.104832$$

$$y(0.6) = 34.43437$$

This is solution of $y(1976) = 34.43437$