Paper 7, TDC Part-3
Chapter– 3, Number Systems and Codes
Electronics
Lecture - 4

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Number Systems and Codes

- **Binary Arithmetic : -**

We perform different types of arithmetic calculation using digital systems like calculator, computer etc. We are also familiar with the things that the data we provide to digital system i.e. inputs, are first converted to binary number then it is handle by the digital system for necessary operation.

Arithmetic operation such as addition, subtraction, multiplication and division is performed to binary numbers in digital systems.

- **Binary Addition : -**

As any number in binary number system is expressed as series combination of 0 & 1, in similar manner the result of addition of two and more numbers in binary system is a series combination of 0 & 1. Therefore the four basic rules for adding 0 & 1 are: -

0 + 0 = 0 ----- Sum part is 0 and carry part is 0.

0 + 1 = 1 ----- Sum part is 1 and carry part is 0.

1 + 0 = 1 ----- Sum part is 1 and carry part is 0.

1 + 1 = 1 0 ----- Sum part is 0 and carry part is 1.

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In the fourth rule we se that addition of two 1’s results in binary two $(10)_2$. When binary numbers are added, the last condition results a sum of ‘0’ in a given column and generate a carry of 1 over to the next column to the left. Let’s see few examples.

Examples- Add the following binary numbers –

a) 11001 and 1000
b) 11100 and 11010
c) 10110101 and 11100011

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Add the following binary numbers:-

(i) $(11001)_2$ and $(1000)_2$

(ii) $(11100)_2$ and $(11010)_2$

(iii) $(10110101)_2$ and $(11100011)_2$

Solution:- (a)

```
     1        → Carry from previous bit
     1 1 0 0 1
 +     1 0 0 0
 -------------
   1 0 0 0 0 1
   └→ Sum
```

As $1 + 1 = 0\ 1$ (Sum Carry). So the generated carry is added to the next higher binary bit.

So Answer is $(100001)_2$

Lets verify this, using decimal number

$(11001)_2 = (25)_{10}$

$+(1000)_2 = +(8)_{10}$

$(100001)_2 = (33)_{10}$

(b)

```
   1 1        Carry
   1 1 1 0 0
 + 1 1 0 1 0
 -----------
 1 1 0 1 1 0
     Sum
```

$1_2 + 1_2 = (10)_2 = (2)_{10}$

$1_2 + 1_2 + 1_2 = (10)_2 + (1)_2$

$= (11)_2 = (3)_{10}$

So answer is $(110110)_2$

Let again verify this using decimal system,

$$
\begin{array}{lcl}
(11100)_2 & = & (28)_{10} \\
+\ (11010)_2 & = & (26)_{10} \\
\hline
(110110)_2 & = & (54)_{10}
\end{array}
$$

(c)

$$
\begin{array}{rcccccccc}
\text{Carry} & 1 & 1 & & 1 & 1 & 1 & 1 & \\
 & & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\
+ & & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\
\hline
\text{Sum} & 11 & 0 & 0 & 1 & 1 & 0 & 0 & 0
\end{array}
$$

$1_2 + 1_2 + 1_2 = (11)_2$

$1_2 + 1_2 + 0_2 = (10)_2$

So answer is $(110011000)_2$.

Let verify this also using decimal system

$$
\begin{array}{lcl}
(10110101)_2 & = & (181)_{10} \\
+\ (11100011)_2 & = & +\ (227)_{10} \\
\hline
(110011000)_2 & = & (408)_{10}
\end{array}
$$

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Example(2) Add the binary numbers as given below.

(i) $(11011)_2$, $(10100)_2$ & $(10001)_2$

(ii) $(110110)_2$, $(100011)_2$, $(001001)_2$ & $(100001)_2$

Soln.- We can add 2 or more binary numbers in two ways.

1st Method :→

1) Take 1st two numbers and add it. Add the third number to the sum obtained by adding the 1st two numbers. Again add the next number to the previous sum obtained and so on.

2) 2nd Method :→

Add all given numbers at a time.

(a) Addition of numbers given above in (i) by 1st method.

$$\begin{array}{r} 11 \\ (11011)_2 \\ +\,(10100)_2 \\ \hline (101111)_2 \end{array}$$

← Sum of 1st two numbers.

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Now adding 3rd number to the sum,

```
           1  1  1  1  1        ← Carries from the previous bit
Sum →     (1  0  1  1  1 | 1 )₂
3rd Number (   1  0  0  0 | 1 )₂
          (1 0  0  0  0  0  0 )₂
```

Students can verify the result in decimal system.

Now, addition of numbers given in (i) through second method.

```
  (1  1  1  1)      ← Carry generated from previous bit
   1  1  0  1  1    ← 1st Number
   1  0  1  0  0    ← 2nd   "
+  1  0  0  0  1    ← 3rd   "
(1 0 0  0  0  0  0)₂
```

b) Addition of ~~give~~ numbers given in (ii) through 1st method.

```
     (1  1)              ← Carry
  1  1  0  1  1  0       ← 1st Number
+ 1  0  0  1  1  1       ← 2nd   "
(1 0  1  1  1  0  1)₂    ← Sum of 1st two number
```

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```
 (1 1 1     1)          ← Carry
  1 0 1 1 1 0 1         ← Sum
+   0 0 1 1 0 1         ← Third Number
(1 1 0 1 0 1 0)₂        ← Sum after adding 3rd number

  1
  1 1 0 1 0 1 0         ← Sum
+ 0 1 0 0 1 0 1         ← Fourth Number
(1 0 0 0 1 1 1 1)₂      ← Final Sum/Result.
```

Students can verify the result in decimal system.

* Now, addition of numbers given in (ii) through second method.

```
 (1 1 10   1 1)         ← Carry
  1 1 0  1 1 0          ← 1st Number
  1 0 0  1 1 1          ← 2nd   ,,
  0 0 1  1 0 1          ← 3rd   ,,
+ 1 0 0  1 0 1          ← 4th   ,,
(1 0 0 0 1 1 1 1)₂      ← Final Sum/Result.
```

From the examples it can observed that,

i) If the number of 1’s to be added in a column is even then the sum bit is ‘0’, and if the number of 1’s to be added in a column is odd then the sum bit is ‘0’.

Binary Subtraction: -

The four basic rules for subtracting 0 & 1 are: -

0 - 0 = 0 ----- Difference part is 0 and borrow part is 0.

0 - 1 = 1 ----- Difference part is 1 and borrow part is 1.

1 - 0 = 1 ----- Difference part is 1 and borrow part is 0.

1 - 1 = 0 ----- Difference part is 0 and borrow part is 0.

Borrow is required when Minuend bit is less than the Subtrahend.

In binary system a borrow of 1 from higher bit position is equivalent to increment of 10_2 that is 2_{10} to lower bit.

Let's see few examples.

Examples- Subtract the following binary numbers –

a) 11001 and 1000

b) 11100 and 11010

c) 11100011 and 10110101

Solution:→(a)

```
   1 1 0 0 1
 −   1 0 0 0
 (1 0 0 0 1)₂
```

Borrow is required

(b)

```
   1 1 1 0 0
 − 1 1 0 1 0
 (0 0 0 1 0)₂
```

(c)

```
   1 1 1 0 0 0 1 1
 − 1 0 1 1 0 1 0 1
 (0 0 1 0 1 1 1 0)₂
```

When a 1 is borrowed from the higher bit position then the lower bit is increment by '2' i.e. '10', and again when 1 is borrowed from this '10' then this becomes 1 and the lower bit becomes '10', and so on. in the same manner. borrow propogates to lower bit from higher bit.

Binary Multiplication: -

The four basic rules for multiplying 0 & 1 in binary system is: -

$0 \times 0 = 0$

$0 \times 1 = 0$

$1 \times 0 = 0$

$1 \times 1 = 1$

Multiplication in binary system is performed like decimal system. It involves forming partial products. Shifting each partial product left one place, and then adding all the partial products.

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Example:- Perform the following binary numbers

(a) 1101 × 110 (b) 1100 × 101

(c) 100 × 11

Solution:- (a)

```
        1 1 0 1      ⟶   (13)10
      ×   1 1 0          × (6)10
      ---------
        0 0 0 0
      1 1 0 1
    1 1 0 1
  -------------
  (1 0 0 1 1 1 0)2   ⟹   (78)10
```

(b)

```
      1 1 0 0        ⟶   (12)10
    ×   1 0 1        ⟶   × (5)10
    -----------
      1 1 0 0
    0 0 0 0
  1 1 0 0
  -----------
  1 1 1 1 0 0        ⟶   (60)10
```

(c)

```
      1 0 0          ⟶   (4)10
    ×   1 1          ⟶   × (3)10
    -------
      1 0 0
    1 0 0
    -------
    1 1 0 0          ⟶   (12)10
```

Binary Division: -

Division in binary system is performed by the same procedure as in decimal system.

Some examples to illustrate division in binary number system

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Example :- Divide (a) $(11011)_2$ by $(101)_2$

(b) $(1001100)_2$ by $(1100)_2$

Solution :- (a)

```
         1 0 1            ← Quotient
  101 ) 1 1 0 1 1         ← Dividend
        1 0 1
        ---------
        ⊗ ⊗ 1 1 1
            1 0 1
            -------
            ✗ 1 0         ← Remainder
```

So, when we divide $(27)_{10} = (11011)_2$ by $(5)_{10} = (101)_2$ will give quotient of $(5)_{10} = (101)_2$ and remainder of $(2)_{10} = (10)_2$

(b)

```
            1 1 0                 ← Quotient
  1100 ) 1 0 0 1 1 0 0            ← Dividend
           1 1 0 0
         -----------
         ✗ ⊗ 1 1 1 0
             1 1 0 0
             ---------
             ✗ ✗ 1 0 0            ← Remainder
```

So when we divide $(76)_{10} = (1001100)_2$

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When we divide $(76)_{10}$ = $(1001100)_2$ by $(12)_{10}$=$(1100)_2$ we find quotient as $(6)_{10}$ = $(110)_2$ and remainder as $(4)_{10}$ = $(100)_2$

Thank You