Paper 7, TDC Part-3

Chapter– 3, Number Systems and Codes

Electronics

Lecture - 3

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Number Systems and Codes

- **Signed Binary Numbers : -**

Like decimal number system, binary number system also deals with the signed numbers. As mentioned in first lecture digital circuits understands only two symbols, ‘0’ and ‘1’; so in binary number systems we use ‘0’ and ‘1’ to represent a positive number and negative number.

An additional bit is used with the binary numbers to indicate a positive or negative number, known as the sign bit. This sign bit is placed to the left of the number that is as the most significant bit (MSB).

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For example in a 8-bit signed number, the first seven bits from right to left represents magnitude of the number while the last 8^{th} bit tells the sign of the number.

Example: - 11001111

Represents magnitude of the signed number that is $(1001111)_2 = (69)_{10}$

This '1' tells that the number is -ve signed number

Similarly in '01001111'; $(1001111)_2 = (69)_{10}$ represents the magnitude and the last most significant bit '0' tell that the number is +ve signed number.

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This representation of binary number is known as sign-magnitude representation.

Q1) For the following sign-magnitude binary numbers find the decimal equivalent number.

a) 000100 b) 101110 c) 001110

Solution- (a) In 000100 the sign bit is '0' so the number is +ve and the remaining bits tells the magnitude that is $(00100)_2 = (4)_{10}$.

So $(000100)_2 = (+4)_{10} = (4)_{10}$

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Solution- (b) In 101110 the sign bit is ‘1’ so the number is -ve and the remaining bits tells the magnitude that is $(01110)_2 = (14)_{10}$.

So $(001110)_2 = (-14)_{10}$

(c) In 001110 the sign bit is ‘0’ so the number is +ve and the remaining bits tells the magnitude that is $(01110)_2 = (14)_{10}$.

So $(001110)_2 = (+18)_{10} = (18)_{10}$

Note :- With unsigned binary number we can represent upto larger value as compared to signed binary numbers. For example 4-bit unsigned binary number can represent from 0 to 15 while 4-bit signed binary number can represent from -7 to +7

- **Complements of Binary Numbers: -**

Complements of binary number permits the representation of negative numbers. The arithmetic operation with negative numbers in digital circuit can be performed in simplified manner using complement of the number.

Binary number systems uses two types of complement – the 1's complement and the 2's complement. The 2's complement arithmetic is commonly used in digital system to handle –ve numbers.

- **One's (1's) Complements of Binary Numbers: -**

Binary number systems uses two types of complement – the 1's complement and the 2's complement. The 2's complement arithmetic is commonly used in digital system to handle –ve numbers.

2's complement method also simplify addition and subtraction of positive and negative numbers in digital systems.

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1's Complement ~~R~~ Representation ~~of~~ of Binary Number

The 1's complement of a binary number is obtained by inverting each bit of the binary number, this means that all 0s in a binary number number is replaced by 1s and all 1s of the binary number is replaced by 0s.

Binary Number → 1 1 0 0 1 0 1

1's complement of (1100101) → 0 0 1 1 0 1 0

So for 1's complement ~~st~~ replace all 1's → 0's
and all 0's → 1's

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So "0011010" is 1's complement of "1100101" and "1100101" is 1's complement of "0011010". If one of the number is positive, then ~~the other~~ its ~~num~~ 1's complement is negative with the same magnitude.

Example → (0111) represents $(+7)_{10}$, 1's complement of $(0111)_2$ is $(1000)_2$ represents $(-7)_{10}$.

The simplest way to obtain 1's complement of a binary number with a digital circuit is to use parallel inverters (NOT gate) for each bit of the binary number.

Figure below shows the digital ckt to obtain 1's complement of 4-bit binary number.

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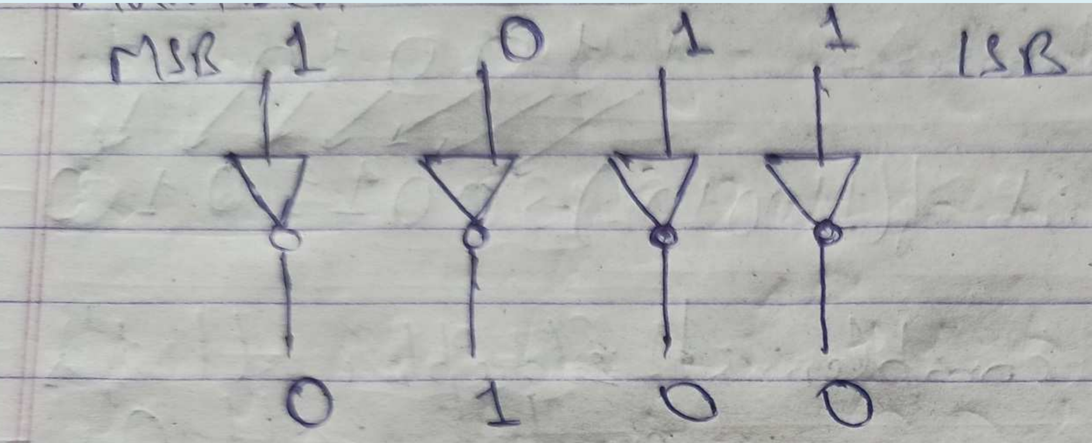

Figure 1 → Digital Circuits to obtain 1's complement of ~~a~~ 4bit -binary number.

Similarly to obtain 1's complement of 8-bit binary number we will increase ~~the~~ 4 more NOT gate. and so on.

Lets see few problems on 1's complement

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Q1) Find the 1's complement of following binary numbers?

a) 110101001 (b) 0010110

(c) 1000111

Soln:- (a) 1's complement of 110101001 → 001010110

(b) 1's complement of 0010110 → 1101001

(c) 1's complement of 1000111 → 0111000

Q2) Represent the following number is 1's complement form.

(a) +24 and -24 (b) +1 and -1

(c) +15 and -15

Number Systems and Codes

(a) +24 and -24 (b) +1 and -1

(c) +15 and -15

1's complement representation.

Soln → (a) $(+24)_{10} = (011000)_2$

$(-24)_{10} = (100111)_2$

(b) $(+1)_{10} = (0001)_2$ in 4-bit

$(-1)_{10} = (1110)_2$ in 4-bit.

(c) $(+15)_{10} = (01111)_2$

$(-15)_{10} = (10000)_2$

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~~From~~ So for n-bit number, the maximum positive number whic can be represented is $(2^{n-1}-1)$ and the max^m^. negative number is $-(2^{n-1}-1)$ in 1's complement.

2's Complement Representation of binary numbers.

2's Complement of a binary number is obtained by adding '1' to the ~~1's~~ ~~complem~~ LSB of 1's complement of the binary number.

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Example :- Find 2's complement of $(11011)_2$

Binary Number → 1 1 0 1 1

1's complement → 0 0 1 0 0

$+ \quad 1$ ← Add

2's Complement 0 0 1 0 1

Find 2's complement of $(011100)_2$

Binary Number → 0 1 1 1 0 0

1's complement → 1 0 0 0 1 1

$+ \quad 1$ ← Add

2's complement 1 0 0 1 0 0

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An alternative direct method to find the 2's complement is :→

1.) Starting from the right that is LSB, copy the bit's as it ~~binary~~ is in the given binary number, upto the 1st '1' encounter.

2.) After that complement (invert) the remaining bits.

Example :→ 1) Binary No. → 1 1 0 1 1 ← 1st One Encounter So Copy it

2's Complement 0 0 1 0 1

→ Complementing remaining bits.

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2) Binary No. → 011100

2's Complement → 100100

Copy the bits Upto this '1'st 1'.

$\underbrace{100}_{\text{Inverting the remaining bits.}}\underbrace{100}_{\text{Copy}}$

) Binary no. → 10000

2's Complement → $\underbrace{10000}_{\text{Copy}}$

Copy the bits Upto this 1'st '1'.

As there is no bits after the 1st '1' encounter so no bits will be inverted.

To convert from a 1's or 2's complement back to the original number, use the same process as used for finding 1's & 2's complement.

For an n-bit number, maximum positive number which can be reperesented in 2's complement form is $(2^{n-1} - 1)$ and the maximum ~~number~~ negative number is -2^{n-1}.

Let us look at the 4-bit binary number representation of some decimal number in sign-magnitude, 1's and 2's complement binary number system in the table

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Decimal Number	4-bit Binary Number		
	Sign-Magnitude Representation	1's Complement	2's Complement
0	0000	0000	0000
1	0001	0001	0001
2	0010	0010	0010
3	0011	0011	0011
4	0100	0100	0100
5	0101	0101	0101
6	0110	0110	0110
7	0111	0111	0111
-8	—	—	1000
-7	1111	1000	1001

-8	—	—	1000
-7	1111	1000	1001
-6	1110	1001	1010
-5	1101	1010	1011
-4	1100	1011	1100
-3	1011	1100	1101
-2	1010	1101	1110
-1	1001	1110	1111
-0	1000	1111	—

Table for different binary numbers.

From table it is clear that using 4-bits in sign magnitude we can represent from -7 to 7, and same is the ~~case~~ range for 1' complement while with 4-bits in 2's

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complement we can represent from
−8 to +7.

In sign magnitude & 1's complement representation '+0' & '−0' can be represented, while with 2's complement representation ~~it is not possible~~ '−0' can not be represented.

Q Represent $(-23)_{10}$ in − (i) sign magnitude (ii) 1's complement (iii) 2's complement.

Solution The minimum number of bits required to represent $(+23)_{10}$ is 6 in signed number system.

$$(+23)_{10} = (\overset{\text{Sign Bit}}{0}\underbrace{10111}_{\text{Magnitude Bit}})_2$$

So (i) Sign Magnitude representation of

$$(-23)_{10} = (110111)_2$$

(ii) 1's Complement representation of $(-23)_{10} = (101000)_2$

(iii) 2's " " " $(-23)_{10} = (101001)_2$

Thank You