Paper 7, TDC Part-3
Chapter– 3, Number Systems and Codes
Electronics
Lecture - 10

By:

Mayank Mausam

Assistant Professor (Guest Faculty)

Department of Electronics

L.S. College, BRA Bihar University,

Muzaffarpur, Bihar

Number Systems and Codes

Example 1) Consider the following 4 codes.

Code A	Code B	Code C	Code D
0001	000	01011	000000
	001		
0010	011	01100	001111
	010		
0100	110	10010	110011
	111		
1000	101	10101	
	100		

Number Systems and Codes

Which of the following properties is satisfied by each of above codes?

(a) Detects Single Error

(b) Detects Double Errors

(c) Detects Triple Errors

(d) Correct Single Error

(e) Correct double errors

(f) Corrects single error and detects double errors.

Soln (a) Minimum distance of Code A → 2

(b) " " " " B → 1

(c) " " " " C → 3

(d) " " " " D → 4

Number Systems and Codes

Now we have formula

"$2c + d + 1$"

for Code A → $2c + d + 1 = 2$

It means we can have $c = 0$ & $d = 1$ such that

$2 \times 0 + 1 + 1 = 2$

So code A can detect ~~error~~ single error and correct no error, because $c = 0$

Number Systems and Codes

→ Hardly for, code B.

$$2c + d + 1 = 1$$

for this $c = 0$ & $d = 0$

This means code B can neither detect any error nor can correct any error

→ For Code C,

$$2c + d + 1 = 3$$

The above condition satisfies for 2 conditions

Number Systems and Codes

Case 1

~~When~~ $c = 0$ & $d = 2$ this means in this case error correction is zero while error detection is 2.

Case 2 : $c = 1$ & $d = 0$ this means in this case error correction is 1 and detection is zero.

> for Code D

$$2c + d + 1 \Rightarrow 4$$

Number Systems and Codes

Case 1 → $C = 0$ & $d = 3$
Case 2 → $C = 1$ & $d = 1$

(a) Detects single error → means $d = 1$ i.e. Code A.

(b) Detects double error → means $d = 2$ i.e. Code C (Case 1)

(c) Detects triple error → means $d = 3$ i.e. Code D (Case 1)

(d) Correct single error → means $C = 1$ i.e. Code E (Case 2)

Number Systems and Codes

(e) Corrects double error i.e. $c = 2$

No code.

(f) Corrects single error and detects double error

$c = 1$ and $d = 2$

No code.

Hamming Code :→

- An error correcting code

Number Systems and Codes

- Constructed by adding k parity bits to each n-bit information.

→ Added in such a way so as to be able to locate the bit position in which error occurs.

→ No. of parity bits depends on the value of n in n-bit information

Such that $2^k \geq n + k + 1$

So for $n = 3$; $k = 3$ so,

$$2^k = 2^3 = 8 \geq n + k + 1$$

$$3 + 3 + 1$$

Number Systems and Codes

for $n = 4$; $k = 3$ so,

$2^3 = 8 \geq n + k + 1 = 4 + 3 + 1 = 8$

- The location of each of the $n+k$ bits within a code word is assigned a decimal number, starting from 1 to the MSB and $(n+k)$ to the LSB.

- The parity bits $p_1, p_2, p_3 \ldots p_k$ are placed in locations $2^0, 2^1, 2^2, \ldots\ldots 2^{k-1}$ i.e.

$1, 2, 4 \ldots\ldots 2^{k-1}$ bits position of the $(n+k)$ code word

Number Systems and Codes

$1, 2, 4 \ldots\ldots 2^{k-1}$ bits position of the $(n+k)$ code word

- The result of each parity check is recorded as '1' if error has been detected and as 0 if no error has been detected.
- k parity checks are performed on selected bits of each code word.

Example (2) Find out the value of k for converting BCD code into Hamming code and the bit positions of the resulting Hamming code.

Soln.

Number Systems and Codes

For a BCD code $n = 4$, therefore k should be such that

$$2^k \geq k + n(=4) + 1)$$

$$2^k \geq k + 5$$

for $k = \cancel{4}\,3$, we have

$$2^3 \geq 3 + 5$$

$$8 = 8$$

Therefore, 3 parity bits are attached to each of the BCD code for constructing the Hamming code. So the BCD Hamming code will be $n+k = 4+3 = 7$ bits with ~~pit~~ bit positions as follows

p_1	p_2	n_1	p_3	n_2	n_3	n_4
1	2	3	4	5	6	7

Based on the parity either even or odd, value assigned to the parity bits ~~so as to mak~~ are '0' or '1'.

In the case of BCD code with 3

Number Systems and Codes

parity bits there are seven error positions. Below table gives these error positions and the corresponding values of the position number.

Error Position	Position Number C_1	C_2	C_3
no error – 0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

Number Systems and Codes

$C_3 = 1$ when error occurs at 1, 3, 5, 7
$C_2 = 1$ " " " " 2, 3, 6, 7
$C_1 = 1$ " " " " 4, 5, 6, 7

Therefore, p_1 is selected so as to establish even (or odd) parity in positions 1, 3, 5, 7

Similarly p_2 & p_3 are selected so as to establish parity in positions 2, 3, 6, 7

Number Systems and Codes

and 4, 5, 6, 7 respectively

(Example 3) Construct Hamming code for BCD 0 1 1 0. Use even parity.

Solution:- For 4-bit code 3 parity bits, p_1, p_2 and p_3 are required i.e. $k = 3$.
Location of parity bits and message bits are as,

	p_1	p_2	m_1	p_3	m_2	m_3	m_4
	1	2	3	4	5	6	7
			0	1	1	1	0
BCD Code			0		1	1	0

Number Systems and Codes

As mentioned in question it is even parity

value of p_1 should be such that there should be even parity (i.e. even no. of '1') at position 1, 3, 5, 7

We have '1' at position 5 (m_2) only so, p_1 = '1' at position 1 will make even parity. So $p_1 = 1$

||arly p_2, should make even parity for position 2, 3, 6, 7

Number Systems and Codes

We have '1' at position 6(m_3) only, so
again p_2 = '1'

For p_3 even parity is required at 4, 5, 6, 7
Ilarly have '1' at position 5(m_2) & 6(m_3)
while '0' at position 7(m_4). So
p_3 = 0 will make even parity for position
(4, 5, 6, 7).

Required Hamming code for BCD digit
0110 with even parity is

"1100110"

Number Systems and Codes

Refer book- Modern Digital Electronics by RP Jain

Thank You