* Newton's General Interpolation with Divided Differences formula:

We have, from divided differences

$$[x, x_0] = \frac{y - y_0}{x - x_0}$$

$$y = y_0 + (x - x_0)[x, x_0] \quad \text{---(i)}$$

Again $$[x, x_0, x_1] = \frac{[x, x_0] - [x_0, x_1]}{x - x_1}$$

$$(x - x_1)[x, x_0, x_1] + [x_0, x_1] = [x, x_0]$$

Substituting this value of $[x, x_0]$ in Eq (i) we then

$$y = y_0 + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x, x_0, x_1] \quad \text{---(ii)}$$

But $$[x, x_0, x_1, x_2] = \frac{[x, x_0, x_1] - [x_0, x_1, x_2]}{x - x_2}$$

$$[x_0, x_1, x_2] = [x, x_0, x_1] + (x - x_2)[x, x_0, x_1, x_2]$$

$$[x, x_0, x_1] = [x_0, x_1, x_2] + (x - x_2)[x, x_0, x_1, x_2]$$

Putting in Eq (ii) then

$$y = y_0 + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] + (x - x_0)(x - x_1)(x - x_2)[x, x_0, x_1, x_2] \quad \text{---(iii)}$$

Proceeding in this way, we obtain:

$$y = y_0 + (x-x_0)[x_0,x_1] + (x-x_0)(x-x_1)[x_0,x_1,x_2] + (x-x_0)(x-x_1)(x-x_2)[x_0,x_1,x_2,x_3] + \cdots$$
$$\cdots + (x-x_0)(x-x_1)\cdots(x-x_n)[x_0,x_1,x_2,\cdots x_n]$$

This formula is called Newton's general interpolation formula with divided differences.

Exp. Find the value at x using the Newton's divided difference formula from the difference table.

Sol. The difference table

$$[x_0,x_1] = \frac{y_1-y_0}{x_1-x_0} = \frac{\Delta y_0}{h} \qquad [x_0,x_1,x_2] = \frac{1}{2h^2}\Delta^2 y_0$$

x	$\log_{10} x = y$	$\Delta y/h$	$\Delta^2 y$
300	2.4771	0.00145	$\Delta y_0/h$
304	2.4829		-0.000001 $\Delta^2 y_0$
		0.00140	
305	2.4843	$\Delta y_1/h$	0 $\Delta^2 y_1$
		0.00140	
307	2.4871	$\Delta y_2/h$	

$x = 301 \Rightarrow \log_{10} 301 = ?$

$x_1 - x_0 = 304 - 300 = 4 = h$ $\Rightarrow [x_0,x_1] = \frac{\Delta y_0}{h} = \frac{y_1-y_0}{h}$

$x_2 - x_0 = 305 - 300 = 5$ $\qquad [x_0,x_1] = \frac{0.00580}{4} = 0.00145$

$$[x_0,x_1,x_2] = \frac{1}{2\times(5)^2}\left[0.00140 - 0.00145\right]$$
$$= \frac{1}{50}\left[-0.00005\right] = -0.000001$$

Using Newton's divided difference formula.

$$y = y_0 + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2]$$

$$\log_{10} 301 = 2.4771 + (301 - 300)(0.00145) + (301 - 300)(301 - 304)(-0.000001)$$

$$= 2.4771 + 0.00145 + 0.000003$$

$$= 2.478553$$

$$\log_{10} 301 = 2.4786$$