# MODULUS OF ELASTICITY

According to Hooke's law, within elastic limit,

$$\text{Stress} \propto \text{Strain}$$

or, $\text{Stress} = E \times \text{Strain}$

or, $\frac{\text{Stress}}{\text{Strain}} = E = \text{a constant}$,

where $E$ is known as coeff. of elasticity or Modulus of elasticity of a body which is independent of magnitude of stress and strain but depends upon the nature of material of the body and the manner in which the body is deformed. Thus modulus of elasticity or coeff. of elasticity of a body is defined as the ratio of the stress to the corresponding strain produced, within the elastic limit.

## Types of modulus of Elasticity

For a given material there can be different types of modulus of elasticity depending upon the type of stress applied and the resulting strain produced. Corresponding to three types of strain, there are three types of modulus of elasticity, as described below:

(1) Young's Modulus of elasticity ($Y$). It is defined as the ratio of normal stress to the longitudinal strain within the elastic limit. Thus

$$Y = \frac{\text{normal stress}}{\text{longitudinal strain}}$$

Consider a metal wire AB of length $l$, radius $r$ and of uniform area of cross-section $a$. Let it be suspended from a rigid support at A, Fig. 1. Let a normal force $F$ be applied at its free end B and let its length increase by $\Delta l$ ($= BB'$)

Then, $\text{longitudinal strain} = \frac{\Delta l}{l}$

$$\text{Normal stress} = \frac{F}{a} = \frac{F}{\pi r^2} \quad (\because a = \pi r^2)$$

$$\therefore Y = \frac{F/\pi r^2}{\Delta l / l} = \frac{Fl}{\pi r^2 \Delta l}$$

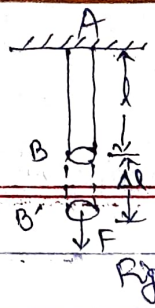


Fig. 1

**Units.** The unit of Young's modulus of elasticity in S.I. is $Nm^{-2}$ or Pascal (denoted by Pa) and in cgs system is dyne/$cm^2$. Its dimensional formula is $[ML^{-1}T^{-2}]$

(2) **Bulk Modulus of elasticity (K).** It is defined as the ratio of normal stress to the volumetric strain, within the elastic limit. Thus

$$K = \frac{\text{normal stress}}{\text{volumetric strain}}$$

Consider a spherical solid body of volume V and surface area a. In order to compress the body, let a force F be applied normally on the entire surface of the body and its volume decreases by $\Delta V$, Fig. 2. Then

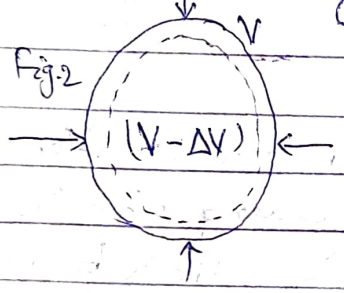


Fig. 2

volumetric strain $= -\Delta V/V$

Here negative sign shows that volume is decreasing when force is applied

$$\text{normal stress} = F/a$$

$$\therefore K = \frac{F/a}{-\Delta V/V} = -\frac{FV}{a\,\Delta V}$$

If p is the increase in pressure applied on the spherical body then $F/a = p$

$$K = -pV/\Delta V$$

**Units.** The units of Bulk Modulus of elasticity in S.I. is $Nm^{-2}$ or Pascal (denoted by Pa) and in cgs system is dyne/$cm^2$. Its dimensional formula is $[ML^{-1}T^{-2}]$.

Compressibility (c) of a material is the reciprocal of its bulk modulus of elasticity (K) i.e.

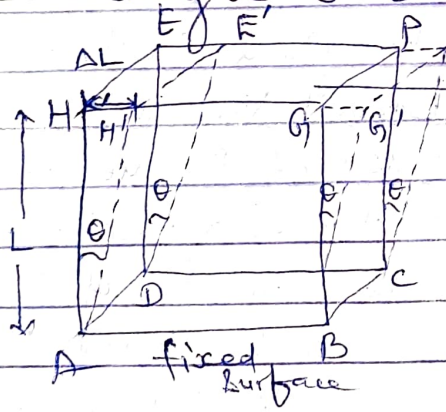
$$C = \frac{1}{K}$$

S.I. units of compressibility is $N^{-1}m^2$ and c.g.s. unit is $dyne^{-1}\,cm^2$.

(B) Modulus of Rigidity ($\eta$). It is defined as the ratio of tangential stress to the shearing strain, within the elastic limit. It is also called shear modulus of rigidity. Thus -

$$\eta = \frac{\text{Tangential Stress}}{\text{Shearing Strain}}$$

Consider a solid metal cube ABCDEFGH whose each side is of length L. Let its lower face ABCD be fixed and a force F be applied tangentially to the top face EFGH, Fig. 3. As a reaction, an equal and opposite force F will be set up on the lower fixed face and a couple will be formed by these forces. Due to this the body will not change its volume but will suffer a change in its shape. Let the vertical planes ADEH and BCFG be shifted laterally to positions ADE'H' and BCF'G', through an angle $\theta$ i.e. $\angle HAH' = \theta = \angle GBG'$.




Let $HH' = \Delta L$

Shearing Strain $= \theta = \frac{HH'}{AH} = \frac{\Delta L}{L}$

Tangential Stress $= F/a$

$$\therefore \eta = \frac{F/a}{\theta} = \frac{F}{a\theta} = \frac{FL}{a\Delta L}$$

units. The unit of $\eta$ in S.I. is $Nm^{-2}$ or Pascal (denoted by Pa) and in cgs system is dyne/cm$^2$. Its dimensional formula is $[ML^{-1}T^{-2}]$.

Elastic After Effect :- We know that on removing the deforming forces from the elastic bodies, they regain their original configuration. Some elastic bodies regain their original configuration immediately, but a few others take some time to recover their original configurations. This temporary delay in

regaining the original configuration of an elastic body after the removal of a deforming force is called elastic after effect. The elastic after effect is negligibly small for quartz and phosphor bronze, but very large for glass fibre. It is due to this reason that the suspensions, made from quartz or phosphorbronze are used in moving coil galvanometers.

Elastic fatigue:-

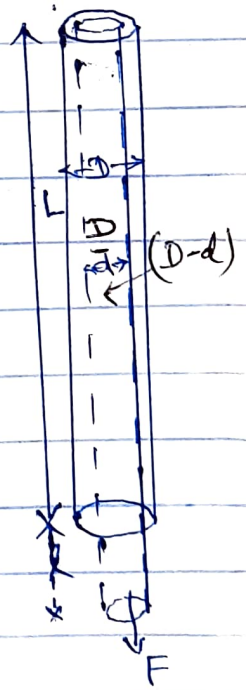
When an elastic body is subjected repeatedly to the action of alternating deforming forces, its behaviour corresponds to that of less elastic bodies. For example, suppose we have two exactly identical wires A and B, fixed to a rigid support and carrying two similar discs at their free ends. Let the disc attached to A be given repeated torsional vibrations about the wire for a few days, whereas the disc attached to B is kept at rest. Now let both the discs be given similar vibrations simultaneously. It is observed that the vibrations of B last longer, whereas that of A die soon. Clearly, the wire A must have been tired or fatigued because of its vibrations prior to the vibrations of B.

Hence elastic fatigue is the property of an elastic body by virtue of which its behaviour becomes less elastic under the action of repeated alternating deforming forces.

It may be interest to note that elastic fatigue corresponds to the physical fatigue in human beings. Just as a tired person is normal after taking due rest, the elastic body is also relieved of the fatigue or regains its original degree of elasticity, when allowed to rest for some time.

# Poisson's Ratio:

When a force acts upon a body along any direction, the size of the body changes not only in that direction but in other directions also. If the force produces elongation in its own direction a contraction also occurs in a direction perpendicular to it i.e. in a lateral direction or vice versa. The change in dimension per unit dimension along which the force acts is called the longitudinal strain while the change in lateral dimension per unit lateral dimension is called lateral strain. Within elastic limits the ratio of the lateral strain to the longitudinal strain is a constant for the material of the body and is known as Poisson's ratio and is denoted by $\sigma$.




If we consider a wire of original length L and diameter D subjected to a tensile stress in the direction shown, then the length of the wire increases from L to (L+l) in the direction of the applied force while the diameter (lateral section) decreases from D to (D-d).

Then the longitudinal strain = $l/L$

and lateral strain = $d/D$

And therefore, the Poisson's ratio of the material of the wire is

$$\sigma = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}} = -\frac{d/D}{l/L} = -\frac{Ld}{Dl}$$

P.T.O.

$$\sigma = \frac{-Ld}{Dl}$$

(negative sign being taken as longitudinal and lateral strain in opposite sense, i.e. if one is positive, another is negative).

If we express Poisson's ratio as a differential coefficient, then

$$\sigma = -\frac{L}{D}\frac{dD}{dL}$$

Poisson's ratio a dimensionless quantity & has no unit.