

(9)

Thus rank $A = \text{rank } [AB]$
and hence the system of equations is
consistent.

Now, the reduced system (1) is

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{or, } \begin{aligned} x - 3y &= 0 \\ y &= 1 \\ -z &= 2 \end{aligned}$$

$$\text{or, } x = 3, y = 1, z = -2$$

This is the required solution.

1 Example (ii) solve $\left. \begin{aligned} 2x - y + 3z &= 9 \\ x + y + z &= 6 \\ x - y + z &= 2 \end{aligned} \right\} \text{(I)}$

Ans:- Matrix method (Gauss method):-

Putting $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \text{ (say)}$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

Then (I) can be written in the matrix form $AX = B$ (I')

$$\text{Now } |A| = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= 2(1+1) + 1(1-1) + 3(-1-1)$$

$$= -2 \neq 0$$

$\therefore A^{-1}$ exists.

Let A_1, A_2 etc. be co-factors of a_1, a_2 etc. in $|A|$.

$$\text{Then, } A_1 = + \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2, \quad A_2 = - \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$A_3 = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2$$

$$B_1 = - \begin{vmatrix} -1 & 3 \\ -1 & 1 \end{vmatrix} = -2, \quad B_2 = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}, \quad B_3 = - \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = 1$$

$$C_1 = \begin{vmatrix} -1 & 3 \\ 1 & 1 \end{vmatrix} = -4, \quad C_2 = - \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 1$$

$$C_3 = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 3$$

$$\therefore A^{-1} = \frac{1}{|A|} \begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 2 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

Now from (I') we have $X = A^{-1}B$

$$\text{i.e. } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix} = \begin{bmatrix} -9 + 6 + 4 \\ 0 + \frac{6}{2} - 3 \\ 9 - 3 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \therefore \begin{aligned} x &= 1 \\ y &= 2 \\ z &= 3 \end{aligned}$$

This is the required solution.

[Note:- In order to show that the equations are consistent, we proceed as follows:-

$$\text{we have } |A| = -2 \neq 0$$

$$\therefore \text{rank } A = 3$$

Also $[AB] = \begin{bmatrix} 2 & -1 & 3 & 9 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \end{bmatrix}$ This is a 3×4 matrix. The highest available order of a minor of

$[AB]$ is 3. Now a third order minor of $[AB]$ is $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \neq 0 \therefore \text{rank } [AB] = 3$
 $= \text{rank } A$

\therefore The equations are consistent.]