Paper 7, TDC Part-3
Chapter– 1, Fundamental Concept of Digital Electronics
Lecture - 6

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Fundamental Concepts of Digital Electronics

- **BOOLEAN Algebra :-**

As a digital signal is discrete in nature and in the digital system these signals assume only one of the two values 0 or 1 at any time.

❖ A number system based on these two digits ‘0’ and ‘1’ is known as “Binary Number System”.

❖ For manipulations of binary number, George Boole developed rules in the middle of the 19th century, known as “Boolean Algebra”. This Boolean algebra is the basis of all digital systems.

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Binary "Variables" can be represented by an English alphabet letter such as A, B, C etc. The Variable can have only one of the two possible values either '0' or '1' at any time.

The "Complement" is the inverse of a variable and is indicated by a bar over the variable (overbar) or by a prime symbol.

Complement of A is $\overline{A}$ or A^{I}

Literal is a variable or the complement of a variable like A, A^{I}, B, B^{I}, X^{I}, Y, Z etc.

Boolean Algebra :-

Boolean Addition rule is equivalent to OR operation

$0 + 0 = 0$

$0 + 1 = 1$ or $1 + 0 = 1$

$1 + 1 = 1$

Boolean Multiplication rule is equivalent to AND operation

$0 . 0 = 0$

$0 . 1 = 0$ or $1 . 0 = 0$

$1 . 1 = 1$

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Boolean Algebraic Theorem :-

1. Theorem No. 1.1) A + 0 = A

Proof : When A = 0 then A + 0 = 0 + 0 = 0

When A = 1 then A + 0 = 1 + 0 = 1

So A + 0 = A

2. Theorem No. 1.1) A . 1 = A

Proof : When A = 0 then A . 1 = 0 . 1 = 0

When A = 1 then A . 1 = 1 . 1 = 1

So A . 1 = A

Theorem 1.1 and 1.2 are called “Duals”

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3. Theorem No. 1.3) A + 1 = 1

Proof : When A = 0 then A + 1 = 0 + 1 = 1

When A = 1 then A + 1 = 1 + 1 = 1

So A + 0 = 1

4. Theorem No. 1.4) A . 0 = 0

Proof : When A = 0 then A . 0 = 0 . 0 = 0

When A = 1 then A . 1 = 1 . 0 = 0

So A . 0 = 0

Theorem 1.3 and 1.4 are called “Duals”

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5. Theorem No. 1.5) A + A = A

Proof : When A = 0 then 0 + 0 = 0 + 0 = 0

When A = 1 then 1 + 1 = 1 + 1 = 1

So A + A = A

6. Theorem No. 1.6) A . A = A

Proof : When A = 0 then A . 0 = 0 . 0 = 0

When A = 1 then A . 1 = 1 . 0 = 0

So A . 0 = 0

Theorem 1.5 and 1.6 are called “Duals”

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7. Theorem No. 1.7) $A + \overline{A} = 1$
8. Theorem No. 1.8) $A . \overline{A} = 0$

Theorem 1.7 and 1.8 are called "Duals"

These theorem can be proved in the same way as proved earlier.

Some other theorems are :-

9. Theorem No. 1.9) $A.(B+C) = AB + AC$

The proof of this theorem can be done through truth table.

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A	B	C	(B+C)	A. (B+C)	AB	AC	(AB+AC)
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Truth Table for Theorem A.(B+C) = AB + AC

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So from truth table we can verify Theorem No. 1.9 that is A.(B+C) = AB + AC.

Value in the column A.(B+C) is equivalent to value in the column AB + AC for every combinations of inputs variable. This verifies the theorem.

In a truth table the total number of input combination is 2^n where n is number of variable. So for, n=2 number of input combination is $2^2 = 4$

n=3 number of input combination is $2^3 = 8$

n=4 number of input combination is $2^4 = 16$

In similar manner we can find total number input combinations for ‘n’ numbers of variable

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10.Theorem No. 1.10) A + BC = (A+B) (A+C)

Proof of Theorem 1.10 can be done as follows,

By taking RHS of theorem

(A+B) (A+C) = A.A + A.B + A.C + B.C

= A + A.B + A.C + B.C [From theorem 1.6 we know that A.A =A]

= A (1 + B + C) + B.C [From theorem 1.3 we know that 1 + any literal is 1]

= A.1 + B.C

= A + BC

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11. Theorem No. 1.11) A + A.B = A

Proof of this theorem should be done by student himself.

12. Theorem No. 1.12) A .(A + B) = A

Proof of this theorem should be done by student himself.

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13. Theorem No. 1.13) $A + \bar{A}.B = (A + B)$

Proof, As per theorem number 10, we can extend

LHS, $A + \bar{A}.B = (A + \bar{A})\,(A + B)$

$= 1.\,(A + B)$ [As per theorem no. 7, $A + \bar{A} = 1$]

$= (A + B)$

14. Theorem No. 1.14) $A(\bar{A} + B) = AB$

15. Theorem No. 1.15) $AB + A\bar{B} = A$

16. Theorem No. 1.16) $(A + B).(A + \bar{B}) = A$

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17.) Theorem No. 1.17) $AB + \bar{A}C = (A+C)(\bar{A}+B)$

proof, We will start by taking R.H.S

$(A+C)(\bar{A}+B) = A\bar{A} + \bar{A}C + AB + BC$

$= 0 + \bar{A}C + AB + BC$ $[A\bar{A} = 0]$

$= \bar{A}C + AB + BC$

$= \bar{A}C + AB + (A+\bar{A})BC$ $[(A+\bar{A}) = 1]$

$= \bar{A}C + AB + ABC + \bar{A}BC$

$= AB(1+C) + \bar{A}C(1+B)$

$= AB.1 + \bar{A}C.1$ [As per theorem no. 1.3]

$= AB + \bar{A}C$

= R.H.S. Hence proved

18.) Theorem No. 1.18) $(A+B)(\bar{A}+C) = AC + \bar{A}B$

19) Theorem No 1.19) $AB + \bar{A}C + BC = AB + \bar{A}C$

[illegible] No. 1.20) $(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

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De Morgan's Theorems :-

De Morgan's proposed 2 theorems, that are an important part of Boolean Algebra

DeMorgan's first theorem states that;

The complement of 2 or more ANDed variables is equivalent to the OR of the complements of the individual variables.

Or,

The complement of a product of variables is equal to the sum of the complements of the variables.

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Or,

The complement of a product of variables is equal to the sum of the complements of the variables.

$$\overline{AB} = \overline{A} + \overline{B}$$

llarly. $$\overline{ABC\ldots\ldots N} = \overline{A} + \overline{B} + \overline{C} + \ldots\ldots + \overline{N}$$

De Morgan's 2nd theorem states that;

The complement of 2 or more ORed variables is equivalent to the AND of the complements of the individual variables.

Or,

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The complement of a sum of variables is equal to the product of the complements of the variables.

i.e.,

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

Ilarly,

$$\overline{A+B+C+\cdots+N} = \overline{A} \cdot \overline{B} \cdot \overline{C} \cdots\cdots \overline{N}$$

We have applied De Morgan's theorem during realization of OR gate using NAND gate or realization of AND gate using NOR gate.

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So De Morgan's Theorem is useful in many Boolean Algebra.

Truth Table to Prove De Morgan's Theorem

$$\overline{A+B} = \bar{A}\cdot\bar{B}$$

A	B	$\bar{A}$	$\bar{B}$	$A+B$	$\overline{A+B}$	$\bar{A}\cdot\bar{B}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

So from truth table we can see that values in column $\overline{A+B}$ equal to values in column $\bar{A}\cdot\bar{B}$.

$$\overline{A+B} = \bar{A}\cdot\bar{B}$$

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A logic problem can be specified in terms of a set of statements. This set of statements can be represented in terms of an equation called the logic equation or in terms of truth table. A digital circuit uses logic gates for the implementation can realise a logic equation.

Using the different theorem, logic equation can be minimise (simplify). The simplified logic equation need less number of gates and/or less number of inputs for the gates.

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Example) Simplify and realise (design) a digital circuit for the logic equation.

$$Y = \bar{A} \cdot B + A \cdot \bar{B} + \bar{A} \cdot \bar{B}$$

Soln. Simplifying

$$
\begin{aligned}
Y &= \bar{A}B + A\bar{B} + \bar{A}\bar{B} \\
&= \bar{A}B + \bar{B}(A + \bar{A}) \\
&= \bar{A}B + \bar{B} \cdot 1 \\
&= \bar{A}B + \bar{B} \\
&= \bar{A} + \bar{B}
\end{aligned}
$$

[As per theorem no. 13].

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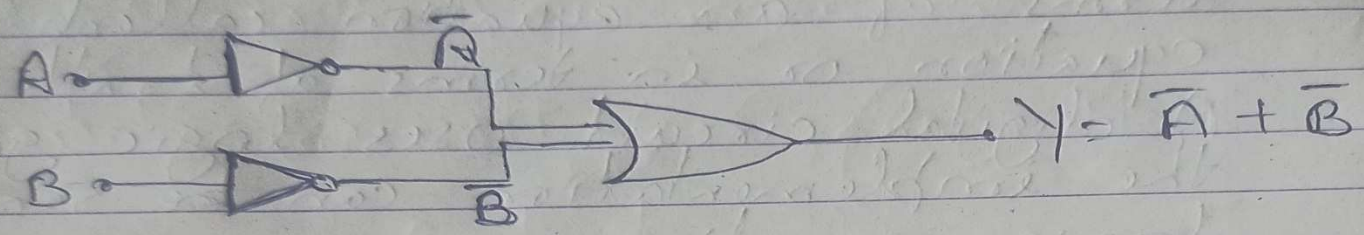
Design using basic gates

$Y = \bar{A} + \bar{B}$

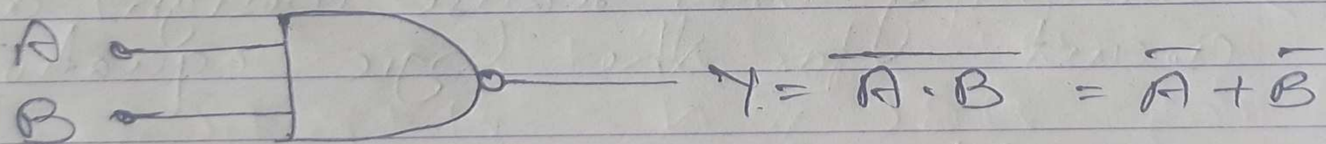


Design using NAND gates only.

$Y = \bar{A} + \bar{B}$

$Y = \overline{\overline{\bar{A} + \bar{B}}} = \overline{\bar{\bar{A}} \cdot \bar{\bar{B}}}$ [As pro De Morgan's theorem]

$= \overline{A \cdot B}$



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Integrated Circuit (IC) Gates

All the logic function (Gates) discussed are commercially available in 'IC' form.

For Example, IC-7400 is a quadruple 2-I/P NAND gate available in 14-pin DIP. This mean IC-7400 contain 4- 2-I/P NAND gate.

Figure below shows Block Diagram of 7400 IC

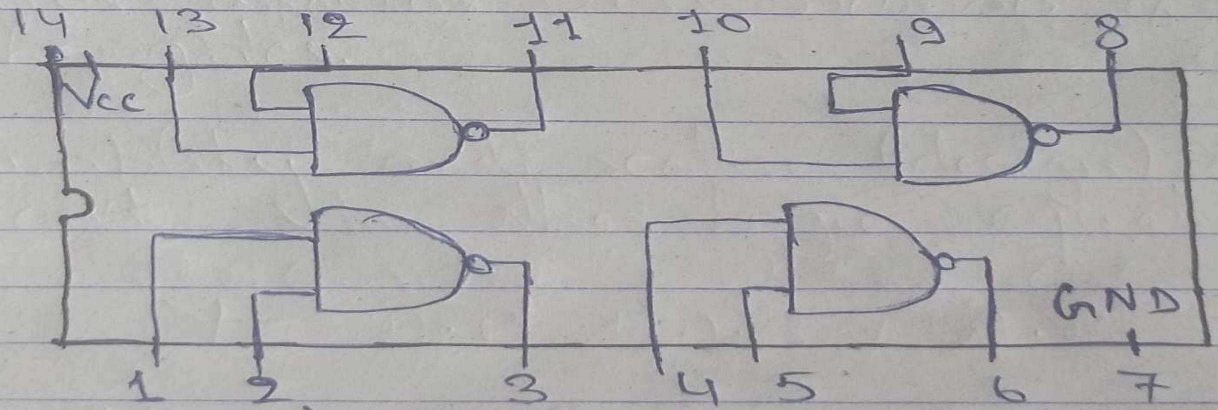

figure 1.1) Block Diagram of 7400 IC

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List of other ICs that contains gates like NOT, OR, AND, XOR etc is provided in the next slide. Different Cs with different number of gates, types of gate or with different numbers of input are available. As per the requirement we use that type of ICs.

The companies that manufacture these are: -

Texas Instruments

Philips

Fairchild Semiconductor

Motorola

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IC No.	Description
7400	Quad 2-input NAND gates
7402	Quad 2-input NOR gates
7404	Hex inverters
7408	Quad 2-input AND gates
7410	Triple 3-input NAND gates
7411	Triple 3-input AND gates
7420	Dual 4-input NAND gates
7421	Dual 4-input AND gates
7427	Triple 3-input NOR gates
7430	8-input NAND gate
7432	Quad 2-input OR gates
7486, 74386	Quad EX-OR gates
74133	13-input NAND gate
74135	Quad EX-OR/NOR gates
74260	Dual 5-input NOR gates