Q. Show that the curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 = 2$ cut orthogonally.

Sol. We know, if two curves cut each other orthogonally then the condition for this is

$$1 + m_1 m_2 = 0$$

$m_1 = \frac{dy}{dx}$ for one curve

$m_2 = \frac{dy}{dx}$ for other curve

Given curves

$$x^3 - 3xy^2 + 2 = 0 \quad \text{--- (I)}$$

$$3x^2y - y^3 - 2 = 0 \quad \text{--- (II)}$$

on differentiating Eqn (I) and (II) w.r.t. x we get

$$3x^2 - (3y^2 + \cancel{3x \cdot 2y} \cdot \frac{dy}{dx}) + 0 = 0 \Rightarrow 3x^2 - 3y^2 - 3 \times 2xy \frac{dy}{dx} = 0$$

$$x^2 - y^2 - 2xy\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{x}{y} - \frac{y}{x} = m_1$$

$$\frac{dy}{dx} = \frac{1}{2xy}(x^2 - y^2) = m_1$$

Similarly.

$$3x^2y - y^3 - 2 = 0$$

$$3 \cdot 2xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} - 0 = 0$$

$$2xy + \frac{dy}{dx}(x^2 - y^2) = 0$$

$$\frac{dy}{dx} = -\frac{2xy}{x^2 - y^2} = m_2$$

Then $1 + m_1 m_2 = 0$

$$1 + \frac{1}{\cancel{2xy}}\cancel{(x^2 - y^2)}\left(-\frac{\cancel{2xy}}{\cancel{x^2 - y^2}}\right)$$

$$\Rightarrow 1 - 1 = 0$$

Hence. These curves cut orthogonally

Q.9 Find where tangent is parallel to the axis of x for the following curves.

(i) $y = (x-1)(x-2)(x-3) \Rightarrow (x^2-3x+2)(x-3)$

$x^3 - 3x^2 + 2x - 3x^2 + 9x$

(v) $3b^2y = x^3 - 3ax^2$

$x^3 - 8x^2 + 17x - 6$

Sol. Given curve (i) $y = (x-1)(x-2)(x-5)$

$$\frac{dy}{dx} = (x-2)(x-3) + (x-1)(x-3) + (x-1)(x-2)$$

$$= x^2 - 5x + 6 + x^2 - 4x + 3 + x^2 - 3x + 2$$

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$$\frac{dy}{dx} = 3x^2 - 12x + 17$$

The Eqn of tangent at $p(x,y)$ and its $\parallel$ to x axis then

$$\frac{dy}{dx} = 0$$

$$\Rightarrow 3x^2 - 16x + 19 = 0$$

$$ax^2 + bx + c \qquad \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 3,\ b = -16,\ c = 19$$

$$x = \frac{(+12) \pm \sqrt{144 - 4 \times 3 \times 17}}{2 \times 3} = \frac{12 \pm \sqrt{12}}{6}$$

$$x = \frac{12 \pm 2\sqrt{3}}{6} = 2 \pm \frac{\sqrt{3}}{3} = 2 \pm \frac{1}{\sqrt{3}}$$