09/04/2021 TDC-I

Indeterminate form $[\frac{\infty}{\infty}]$

Q.7

Exp. Evaluate $\lim_{x\to 0} \log_x \tan x$

Solution $\lim_{x\to 0} \log_x \tan x$

we have $\because \log_a m = \frac{\log_e m}{\log_e a}$

$\lim_{x\to 0} \frac{\log_e \tan x}{\log_e x}$

$\lim_{x\to 0} \frac{\log \tan x}{\log x}$ This is form $\frac{\infty}{\infty}$

Then by L'Hospital rule, the given lim

$$\lim_{x\to 0} \frac{\frac{d}{dx}(\log_e \tan x)}{\frac{d}{dx}(\log_e x)} = \lim_{x\to 0} \frac{(1/\tan x)\cdot \sec^2 x}{1/x}$$

$$= \lim_{x\to 0} \frac{x \cdot \sec^2 x}{\tan x}$$

$$= \lim_{x\to 0} \frac{x \cdot \cos x}{\sin x \cdot \cos^2 x}$$

$$= \lim_{x\to 0} \frac{x \cdot 1}{\sin x \cdot \cos x}$$

$$= \lim_{x\to 0} \frac{2x}{\sin 2x}$$ Again L'HR

$$= \lim_{x\to 0} \frac{2\cdot 1}{\cos 2x \cdot 2} = \lim_{x\to 0} \frac{1}{\cos 2x}$$

$$= \frac{1}{\cos 0} = \frac{1}{1}$$

$$\lim_{x\to 0} \log_x \tan x = 1$$

Q.9 Evaluate $\lim_{x\to 0} \log_{\tan^2 x}(\tan^2 2x)$

Solution $\lim_{x\to 0} \log_{\tan^2 x}(\tan^2 2x)$

$$= \lim_{x\to 0} \frac{\log_e \tan^2 2x}{\log_e \tan^2 x} \quad \because \log_n m = \frac{\log_e m}{\log_e n}$$

$$= \lim_{x\to 0} \frac{2\log_e \tan 2x}{2\log_e \tan x} \quad \because \log_e x^2 = 2\log_e x$$

$$= \lim_{x\to 0} \frac{\log_e \tan 2x}{\log_e \tan x}$$ This is form $\frac{\infty}{\infty}$

$$= \lim_{x\to 0} \frac{\frac{1}{\tan 2x}\cdot \sec^2 2x \cdot 2}{\frac{1}{\tan x}\cdot \sec^2 x}$$ applying L.H.Rule

$$= \lim_{x\to 0} \frac{\frac{\cos 2x}{\sin 2x}\cdot \frac{2}{\cos^2 2x}}{\frac{\cos x}{\sin x}\cdot \frac{1}{\cos^2 x}}$$

$$= \lim_{x\to 0} \frac{2/\sin 2x \cdot \cos 2x}{1/\sin x \cdot \cos x}$$

$$= \lim_{x\to 0} \frac{2\sin x \cdot \cos x}{\sin 2x \cdot \cos 2x}$$

$$= \lim_{x\to 0} \frac{\sin 2x}{\sin 2x \cdot \cos 2x}$$

$$= \lim_{x\to 0} \frac{1}{\cos 2x} = \frac{1}{\cos 0} = \frac{1}{1} = 1$$

$\therefore \lim_{x\to 0} \log_{\tan^2 x} \tan^2 2x = 1$