Exp. In the catenary $y = a\cosh\frac{x}{a}$, prove that the length of the portion of the normal intercepted between the curve and the x-axis as y^2?

Sol. Give catenary $y = a\cosh\frac{x}{a}$

Diff. w.r.to x, we get

$$\frac{dy}{dx} = \cancel{a}\cdot \sinh\frac{x}{a}\cdot\frac{1}{\cancel{a}} = \sinh\frac{x}{a}$$

Now, the length of the normal $= y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

$$= a\cosh\frac{x}{a}\cdot\sqrt{1+\left(\sinh\frac{x}{a}\right)^2}$$

$$= a\cosh\frac{x}{a}\cdot\sqrt{\cosh^2\frac{x}{a}}$$

$$= a\cosh\frac{x}{a}\cdot\cosh\frac{x}{a}$$

$$= a\cosh^2\frac{x}{a}$$

$\because y = a\cosh\frac{x}{a}$

$\Rightarrow \cosh\frac{x}{a} = \frac{y}{a}$

$$= \cancel{a}\cdot\frac{y^2}{a^{\cancel{2}}}$$

$$= \frac{y^2}{a} \Rightarrow \frac{1}{a}\cdot y^2$$

$\therefore$ normal $\propto y^2$

Q.7. Show that in the curve $y = a\log(x^2 - a^2)$ the sum of the tangent and subtangent varies as the product of the co-ordinates of the point.

<u>Sol.</u> Given the curve $y = a\log(x^2 - a^2)$

Diff. w.r.t. x, we get

$$\frac{dy}{dx} = a \cdot \frac{1}{x^2 - a^2} \cdot 2x = \frac{2ax}{x^2 - a^2}$$

Now, the length of the tangent $= \dfrac{y \cdot \sqrt{1 + (dy/dx)^2}}{dy/dx}$

$$T = \frac{y \cdot \sqrt{1 + \left(\frac{2ax}{x^2 - a^2}\right)^2}}{\frac{2ax}{x^2 - a^2}}$$

$$T = \frac{y \cdot \sqrt{(x^2 - a^2)^2 + 4a^2x^2}}{2ax \cdot \cancel{(x^2 - a^2)}} \cancel{(x^2 - a^2)}$$

$$T = \frac{y \cdot \sqrt{x^4 + a^4 - 2a^2x^2 + 4a^2x^2}}{2ax}$$

$$T = \frac{y \cdot \sqrt{x^4 + a^4 + 2a^2x^2}}{2ax} = \frac{y \cdot \sqrt{(x^2 + a^2)^2}}{2ax}$$

$$T = \frac{y\,(x^2 + a^2)}{2ax} \quad \text{--- (1)}$$

And the length of the subtangent $= y / dy/dx$

$$ST = \frac{y}{2ax/x^2 a^2}$$

$$ST = \frac{y(x^2 a^2)}{2ax} \quad \text{---} \quad (11)$$

The sum of tangent (T) and subtangent (ST)

$$= \frac{y(x^2 + a^2)}{2ax} + \frac{y(x^2 a^2)}{2ax}$$

$$= \frac{y}{2ax}\left[x^2 + \cancel{a^2} + x^2 - \cancel{a^2}\right]$$

$$= \frac{\cancel{2}x^{\cancel{2}}y}{\cancel{2}a\cancel{x}} = \frac{1}{a}xy$$

$$T + ST = \frac{1}{a}xy$$

$$(T + ST) \propto xy$$

Hence sum of tangent and sub tangent varies as the product of the ~~co-ordi~~ coordinate (x, y).

— x —