Exp. Evaluate:

$$I = \int_0^1 \frac{1}{1+x}\,dx$$ correct to three decimal places.

Solution This example is solved by both the trapezoidal and Simpson's 1/3-rules with $h=0.5$ and $h=0.25$ respectively. $x_0=0$ & $x_n=1$

(i) for $h=0.5$ The values of x and y are

x	$y=\frac{1}{1+x}$	
0.0	1.0	y_0
0.5	0.6667	y_1
1.0	0.5000	$y_2 \to y_n$ $n=2$

(a) Trapezoidal rule gives

$$I = \frac{h}{2}[y_0 + 2 \cdot y_1 + y_2]$$

$$I = \frac{0.5}{2}[1.0 + 2\times 0.6667 + 0.5]$$

$$I = 0.25[1.5 + 1.3334] = 0.70835$$

(b) Simpson's $\frac{1}{3}$ rule gives

$$I = \frac{h}{3}[y_0 + 4y_1 + y_2]$$

$$= \frac{0.5}{3}[1.00 + 4\times 0.6667 + 0.5] =$$

$$= \frac{0.5}{3}[1.5 + 2.6668]$$

$$= \frac{0.5 \times 4.1668}{3} = 0.69446$$

$$= 0.6945$$

$$= 0.6945$$

(ii) For $h = 0.25$, The values of x and y are as

x	$y = \frac{1}{1+x}$	
0.00	1.0000	y_0
0.25	0.8000	y_1
0.50	0.6667	y_2
0.75	0.5714	y_3
1.00	0.5000	$y_4 \rightarrow y_n$

$n = 4$

(a) Trapezoidal rule gives

$$I = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3) + y_4\right]$$

$$= \frac{0.25}{2}\left[1.00 + 2(.8000 + 0.6667 + 0.5714) + 0.5\right]$$

$$= 0.125\left[1.5 + 2 \times 2.0381\right]$$

$$= 0.125\left[5.5762\right]$$

$$= 0.6970$$

(b) Simpson's rule gives.

$$I = \frac{h}{3}\left[y_0 + 4(y_1 + y_3) + 2(y_2) + y_4\right]$$

$$= \frac{0.25}{3}\left[1.0000 + 4(.8000 + 0.5714) + 2(0.6667) + 0.5000\right]$$

$$= \frac{0.25}{3}\left[1.5 + 4 \times 1.3714 + 2 \times 0.6667\right]$$

$$= \frac{0.25}{3}\left[1.5 + 5.4856 + 1.3334\right]$$

$$= 0.69325$$

(iii) for $h = 0.125$, then the values of x and y are

x	$y=\frac{1}{1+x}$	
0.000	1.0000	y_0
0.125	0.8889	y_1
0.250	0.8000	y_2
0.375	0.7273	y_3
0.500	0.6667	y_4
0.625	0.6154	y_5
0.750	0.5714	y_6
0.875	0.5333	y_7
1.000	0.5000	$y_8 \longrightarrow y_n \quad n=8$

(a) Trapezoidal rule gives

$$I = \frac{h}{2}\left[y_0 + 2(y_1+y_2+y_3+y_4+y_5+y_6+y_7) + y_8\right]$$

$$I = \frac{0.125}{2}\left[1.00 + 2(0.8889+0.8000+0.7273+0.6667+0.6154+0.5714+0.5333) + .500\right]$$

$$I = 0.6941$$

(b) Simpson's rule gives,

$$I = \frac{h}{3}\left[y_0 + 4(y_1+y_3+y_5+y_7) + 2(y_2+y_4+y_6) + y_8\right]$$

$$= \frac{0.125}{3}\left[1.00 + 4(0.8889+0.7273+0.6154+0.5333) + 2(0.8000+0.6667+0.5714) + 0.500\right]$$

$$= \frac{0.125}{3}\left[1.5 + 4\times(2.7649) + 2(2.0381)\right]$$

$$= \frac{0.125}{3}\left[1.5 + 11.0596 + 4.0762\right]$$

$$= 0.6932$$

Exp. Calculate the value of $I = \int_0^{\pi/2} \sqrt{\sin x}\, dx$ using Simpson's $\frac{1}{3}$ rule.

Sol^n:- Let $n = 6 \Rightarrow nh = b - a \Rightarrow h = \frac{\pi/2 - 0}{6} = \pi/12$

$y = \sqrt{\sin x}$ Then value of x and y are

x	$y = \sqrt{\sin x}$	y
$x_0 = 0$	$y_0 = \sqrt{\sin 0} = \sqrt{\sin 0^\circ}$	$= 0$
$x_1 = \pi/12$	$y_1 = \sqrt{\sin \pi/12} = \sqrt{\sin 15^\circ} =$	0.5087
$x_2 = 2\pi/12$	$y_2 = \sqrt{\sin 2\pi/12} = \sqrt{\sin 30^\circ} =$	0.7071
$x_3 = 3\pi/12$	$y_3 = \sqrt{\sin 3\pi/12} = \sqrt{\sin 45^\circ}$	$= 0.8409$
$x_4 = 4\pi/12$	$y_4 = \sqrt{\sin 4\pi/12}$ $\sqrt{\sin 60^\circ}$	$= 0.9306$
$x_5 = 5\pi/12$	$y_5 = \sqrt{\sin 5\pi/12}$ $\sqrt{\sin 75^\circ}$	$= 0.9825$
$x_6 = 6\pi/12$	$y_6 = \sqrt{\sin 6\pi/12}$ $\sqrt{\sin 90^\circ}$	$= 1.0000$

Using Simpson's $\frac{1}{3}$ rule:

$$I = \int_0^{\pi/2} \sqrt{\sin x}\, dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) + y_6\right]$$

$$= \frac{1}{3} \cdot \frac{\pi}{12}\left[0 + 4(0.5087 + 0.8409 + 0.9825) + 2(0.7071 + 0.9306) + 1\right]$$

$$= \frac{1}{\cancel{36}_{18}} \times \frac{\cancel{22}^{11}}{7}\left[0 + 4 \times 2.3321 + 2 \times 1.6377 + 1\right]$$

$$= \frac{11}{18 \times 7}\left[1 + 9.3284 + 3.2754\right]$$

$$= \frac{11 \times 13.6038}{18 \times 7} = \frac{149.6418}{18 \times 7}$$

$$= 1.1876$$