Exp. Given that $\frac{dy}{dx} = y - x$ where $y(0) = 2$ find $y(0.1)$ and $y(0.2)$ correct to four decimal places.

Solution (i) Runge-Kutta second order formula

$$y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$$

We find k_1 and k_2 from $h = 0.1$ and $f(x,y) = y - x$, $x_0 = 0$ & $y_0 = 2$ then

$$k_1 = hf(x_0, y_0)$$

$f(x,y) = y - x$ given

then

$$f(x_0, y_0) = y_0 - x_0 = 2 - 0 = 2$$

$$k_1 = 0.1(2) = 0.2$$

$$k_2 = hf(x_0 + h, y_0 + k_1)$$

$$f(x_0 + h, y_0 + k_1) = (y_0 + k_1) - (x_0 + h)$$

$$= (2 + 0.2) - (0 + 0.1)$$

$$k_2 = 2.2 - 0.1 = 2.1$$

Then $y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$

$y_1 = y(0.1) = 2 + \frac{1}{2}(0.2 + 2.1) = 2.2$

To determine $y_2 = y(0.2)$

We note that $x_0 = 0.1$ and $y_0 = 2.2$

then $f(x_0, y_0) = y_0 - x_0 = 2.2 - 0.1$

$f(x_0, y_0) = 2.1$

$k_1 = h \cdot f(x_0, y_0)$

$k_1 = 0.1\,(2.105) = 0.2105$

$k_2 = 0.1\left[(2.205 + .2105) - (0.1 + 0.1)\right]$

$k_2 = 0.1\left[2.4155 - 0.2\right] = .22155$

then

$y_2 = y(.2) = y_0 + \frac{1}{2}(k_1 + k_2)$

$= 2.205 + \frac{1}{2}(0.2105 + .22155)$

$= 2.205 + \frac{1}{2}(0.43205)$

$= 2.205 + .216025$

$= 2.421025$

$y_2 = y(0.2) = 2.4210$

Proceeding in a similar way we obtain $y_3(0.3)$ and $y_4(0.4)$ from $x_0 = 0.2$ and $y_0 = 2.4210$

then $f(x_0, y_0) = y_0 - x_0$

$= 2.4210 - .2$

$f(x_0, y_0) = 2.2210$

$k_1 = h \cdot f(x_0, y_0) = 0.1\,(2.2210)$

$k_1 = .22210$

$k_2 = h\left[(y_0 + k_1) - (x_0 + h)\right]$

$= 0.1\left[(2.4210 + .2210) - (.2 + .1)\right]$

$k_2 = 0.1\left[2.6431 - 0.3\right] = .23431$

$y_3 \equiv y(0.3) = y_0 + \frac{1}{2}(k_1 + k_2)$

$= 2.4210 + \frac{1}{2}(.22210 + .23431)$

$= 2.4210 + \frac{1}{2}(.45520)$

$= 2.4210 + .22760$

$y_3 = y(0.3) = 2.6492$

Similarly for $y_4 = y(.4)$ from

$x_0 = 0.3$ & $y_0 = 2.6492$

Same process as proceeding way

$y_4 = y(0.4) = 2.8909$