

Exp. If $y_1 = 4$, $y_3 = 12$, $y_4 = 19$ and $y_x = 7$
find x .

Sol.

	x	y_x	Interchange x and y for inverse interpolation		
	x_0	y_0	x_0	y_0	
	1	4	4	(7)	
	x_1	y_1	x_1	y_1	
	3	12	12	3	
	x_2	y_2	x_2	y_2	
	4	19	19	4	

Given that $y_x = 7 \Rightarrow x = 7$ then find x .

Using inverse interpolation formula.

$$y(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$y(x) = \frac{(7-12)(7-19)}{(4-12)(4-19)} (1) + \frac{(7-4)(7-19)}{(12-4)(12-19)} (3)$$

$$+ \frac{(7-4)(7-12)}{(19-4)(19-12)} (4)$$

$$= \frac{(-5)(-12)}{(-8)(-15)} \times 1 + \frac{(3)(-12)}{(8)(-7)} \times 3 + \frac{(3)(-5)}{(15)(7)} \times 4$$

$$= \frac{1}{2} + \frac{27}{14} - \frac{4}{7}$$

$$y(x) = 1.86$$

This value find at $x=7$ from inverse interpolation formula therefore $x = 1.86$

Exp. Solve the same example by means of Aitken's scheme at $y_0 = 7$

	y_0	x
x_0	4	$1 \rightarrow y_0$
x_1	12	$3 \rightarrow y_1$
x_2	19	$4 \rightarrow y_2$

We know Aitken's scheme for the arguments x_0 and x_j the linear polynomial is

$$\Delta_{0j}(x) = \frac{1}{x_j - x_0} \begin{vmatrix} y_0 & x_0 - x \\ y_j & x_j - x \end{vmatrix}, \quad j=1, 3, \dots, n$$

Similarity

$$\Delta_{01j}^{(2)} = \frac{1}{x_j - x_1} \begin{vmatrix} \Delta_{01}(x) & x_1 - x \\ \Delta_{0j}(x) & x_j - x \end{vmatrix}, \quad j=2, 3, \dots, n$$

and

$$\Delta_{01,3,\dots,k}^{(2)}(x) = \frac{1}{x_j - x_k} \begin{vmatrix} \Delta_{01,2,\dots,k}(x) & x_k - x \\ \Delta_{01,3,\dots,(k-1)j}(x) & x_j - x \end{vmatrix}$$

for $j = k+1, \dots, n$

Thus.

$$\Delta_{01}(x) = \frac{1}{12-4} \begin{vmatrix} 1 & 4-7 \\ 3 & 12-7 \end{vmatrix} = \frac{1}{8} (5+9) = \frac{14}{8} = 1.75$$

$$\Delta_{02}(x) = \frac{1}{19-4} \begin{vmatrix} 1 & 4-7 \\ 4 & 19-7 \end{vmatrix} = \frac{1}{15} (12+12) = \frac{24}{15} = 1.6$$

$$\Delta_{012}(x) = \frac{1}{19-12} \begin{vmatrix} 1.75 & 12-7 \\ 1.6 & 19-7 \end{vmatrix} = \frac{1}{7} (21-8) = 1.857$$

Therefore Aitken's scheme Table

y	x	$\Delta_{0j}(x)$ $j=1,2,\dots$	$\Delta_{0j}(x)$ $j=3,4,\dots$
4	1	1.750	
12	3	1.600	1.857
19	4		

$y_{\text{max}} = 7$ find $x = 1.857$