Eigenvalues:- Let V be a finite dimensional vector space over a field F and T be a linear operator on V i.e $T: V \to V$, then a scalar $\lambda \in F$ is called an eigenvalue of T, if there exists a non-zero vector $v \in V$ such that

$$T(v) = \lambda v \quad \text{where } v \neq 0$$

$v \to$ Eigenvector (non zero vector)
$\lambda \to$ Eigenvalue (characteristic values)

Also each such non-zero vector $v \in V$ is called an eigenvector of T corresponding to λ.

Diagonalization : Let V .... F. Then a linear operator $T: V \to V$ is said to be diagonalization if V has a basis consisting of eigenvectors of T only.

* In other words, A Linear operator T has n linearly independent eigenvectors, if V is n-dimensional vector space.

Exp. A scalar multiple of identity operator is diagonalizable

Exp
Theorem – Let $T: V \to V$ be a linear operator on a finite dimensional vector space over a field F. Then $\lambda \in F$ is an eigenvalue of T iff (if and only if) the operator $(T - \lambda I)$ is singular. The eigenspace of λ is then the Kernel of $(T-\lambda I)$, $\ker(T - \lambda I) = 0$ Singular (not invertible)

Proof Let $\lambda \in F$ is an eigenvalue of T and v be any nonzero vector of V. then by Eigenvalue of T

$$T(v) = \lambda v \quad \text{or} \quad T(v) - \lambda v = 0$$

$$T(v) - \lambda . I(v) = 0 \quad (T - \lambda I)(v) = 0 \quad v \neq 0$$

$T - \lambda I = 0 \quad \because v \neq 0$

$\Rightarrow$ $T - \lambda I$ is singular and hence not invertible.

If $T - \lambda I$ is not invertible (not inverse)

$\Rightarrow \det(T - \lambda I) = 0$

$|(T - \lambda I)| = 0$

<u>Conversly</u>, Let $v \in V \quad v \neq 0$

So it is linearly independent

Since $(T - \lambda I)$ is singular

$\Rightarrow |(T - \lambda I)| = 0$

$\Rightarrow (T - \lambda I)(v) = 0 \quad \because v$ is linearly independe..

$T(v) - \lambda I(v) = 0 \quad \because I \cdot (v) = v$

$T(v) - \lambda v = 0$

$T(v) = \lambda v$

λ is an eigenvalue of T.

Also, $(T - \lambda I)(v) = 0 \Rightarrow v \in$ kernel of $(T - \lambda I)$

In this theorem, λ is an eigenvalue of T iff λ is a root of this characteristic polynomial of T in F.

i.e. $|T - \lambda I| = 0$