(3) Show that a rectangular hyperbola can be described by a particle under a force parallel to an asymptote which varies as the cube of its distance from the other asymptote.

Taking the asymptotes as axes of reference, let the equation to the rectangular hyperbola be

$$xy = c^2$$

$$y = \frac{c^2}{x} \quad \text{——— (1)}$$

Differentiating we get $\frac{dy}{dt} = -\frac{c^2}{x^2}\frac{dx}{dt}$ ——— (2)

If there is no force parallel to x-axis we have

$$\frac{d^2x}{dt^2} = 0 \quad \text{——— (3)}$$

which shows that $\frac{dx}{dt}$ = constant = u (say)

Hence (2) becomes

$$\frac{dy}{dt} = -\frac{c^2}{x^2}u$$

$$= -\frac{u}{c^2}y^2 \quad \text{(by (1))}$$

$$\therefore \frac{d^2y}{dt^2} = -\frac{2u}{c^2}y\frac{dy}{dt} = \frac{2u^2}{c^4}y^3 \propto y^3 \quad \text{——— (4)}$$

(3) and (4) prove the theorem.