B.Sc. I
~~Paper~~-II
G- B.

Derive Planck's Law of black body radiation and discuss its experimental verification.

In 1900, Planck introduced an entirely new idea that the emission of black-body radiation is not continuous, but in the form of energy packets, called 'quanta', each quantum carrying an energy $h\nu$, where ν is the frequency of radiation and h is Planck's constant.

We can now combine Planck's expression for the average energy of an oscillator with the number of oscillators per unit volume of the radiation as computed by Rayleigh and Jeans. Thus, the energy-density of radiation, u_ν, in the frequency range ν to $\nu+d\nu$ is given by

$$u_\nu d\nu = \frac{8\pi\nu^2 d\nu}{c^3} \times \bar{E} \quad \text{——— (1)}$$

The term $\frac{8\pi\nu^2 d\nu}{c^3}$ represents the number of electromagnetic standing waves per unit volume in the frequency range ν to $\nu + d\nu$ in the cavity radiation.

Substituting the value of $\bar{E}$ from given eqn.

$$\bar{E} = \frac{h\nu}{e^{h\nu/KT} - 1}$$ which is average energy of a Planck's oscillator, we have,

$$u_\nu \, d\nu = \frac{8\pi\nu^2 d\nu}{c^3}\left(\frac{h\nu}{e^{h\nu/KT} - 1}\right)$$

or, $$u_\nu \, d\nu = \frac{8\pi h \nu^3}{c^3} \frac{d\nu}{e^{h\nu/KT} - 1} \quad \text{---} (2)$$

This is Planck's radiation formula in terms of frequency ν.

Now we can express it in terms of wavelength, we observe that, since $\nu = \frac{c}{\lambda}$,

$$d\nu = -\frac{c}{\lambda^2} d\lambda$$

and since an increase in frequency corresponds to a decrease in wavelength,

$$u_\lambda \, d\lambda = -u_\nu d\nu.$$

Therefore, $$u_\lambda \, d\lambda = \frac{8\pi h}{c^3} \frac{\left(\frac{c}{\lambda}\right)^3 \left(\frac{c}{\lambda^2} d\lambda\right)}{e^{hc/\lambda KT} - 1}$$

or, $$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda KT} - 1} \quad \text{--- (3)}$$

This is Planck's formula of black body radiation in terms of wavelength.

Experimental verification:

The Planck's formula is found to be in complete agreement with experiment for the entire wavelength range at all temperatures.

Case I : when λ is very small, then $e^{hc/\lambda KT} >> 1$, so that Planck's formula given by eqn (3) can be written as

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda KT} d\lambda.$$

putting $8\pi hc = A$ and $\frac{hc}{k} = B$, we have

$$u_\lambda d\lambda = \frac{A}{\lambda^5} e^{-B/\lambda T} d\lambda.$$

This is Wein's law which agrees with experiment at short wavelengths.

Case II : when λ is very large, then $e^{hc/\lambda KT} \simeq 1 + \frac{hc}{\lambda KT}$, so that eqn

(3) Can be written as

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left(1+\frac{hc}{\lambda KT}-1\right)} d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda .$$

This is Rayleigh - Jeans law which agrees with experiment at long wavelengths.

On the basis of above assumption we can plot a graph as follows :

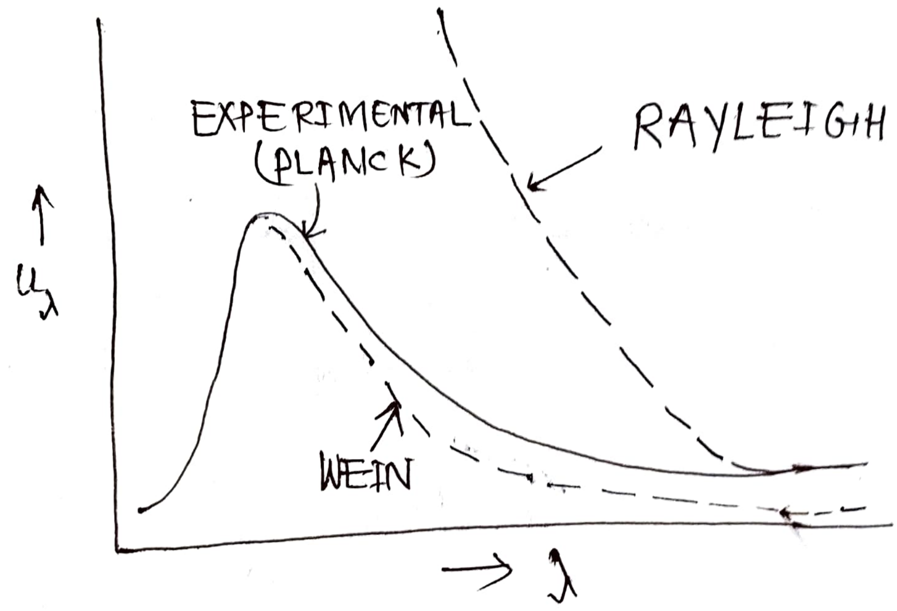

Fig - (1)

———— x ————