9.4.1 Mathematical formulation of second law of motion

Suppose an object of mass, m is moving along a straight line with an initial velocity, u. It is uniformly accelerated to velocity, v in time, t by the application of a constant force, F throughout the time, t. The initial and final momentum of the object will be, $p_1 = mu$ and $p_2 = mv$ respectively.

The change in momentum

$$\propto p_2 - p_1$$
$$\propto mv - mu$$
$$\propto m \times (v - u).$$

The rate of change of momentum $\propto \dfrac{m \times (v-u)}{t}$

Or, the applied force,

$$F \propto \frac{m \times (v-u)}{t}$$

$$F = \frac{km \times (v-u)}{t} \qquad (9.2)$$

$$= kma \qquad (9.3)$$

Here a [$= (v - u)/t$] is the acceleration, which is the rate of change of velocity. The quantity, k is a constant of proportionality. The SI units of mass and acceleration are kg and m s^{-2} respectively. The unit of force is so chosen that the value of the constant, k becomes one. For this, one unit of force is defined as the amount that produces an acceleration of 1 m s^{-2} in an object of 1 kg mass. That is,

1 unit of force = $k \times$ (1 kg) $\times$ (1 m s^{-2}).

Thus, the value of k becomes 1. From Eq. (9.3)

$$F = ma \qquad (9.4)$$

The unit of force is kg m s^{-2} or newton, which has the symbol N. The second law of

motion gives us a method to measure the force acting on an object as a product of its mass and acceleration.