

Paper 1, TDC Part-1
Chapter– 4, Circuit Theorems
Lecture - 10

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Circuit Theorem

In this lecture we will look some problem on 'Reciprocity' Theorem.

Problem based on Reciprocity

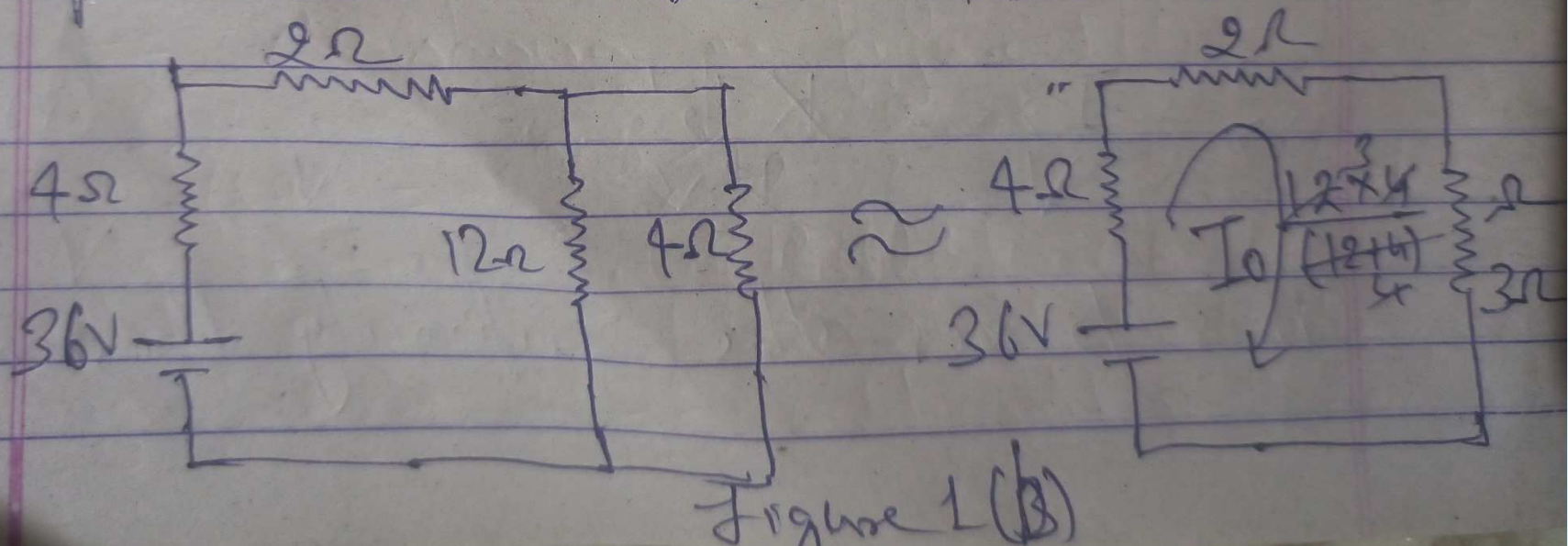
Q1) For the circuit shown in figure 1(a) verify the reciprocity theorem for current in through ~~1Ω resistor~~ branch with 1Ω resistor.

Figure 1(a)

Circuit Theorem

Solution → Let's 1st find current I passing through $1\ \Omega$ resistor.

We can redraw the circuit of figure 1(a) as below where series combination of $3\ \Omega$ & $1\ \Omega$ resistor can be replaced by $4\ \Omega$ resistor parallel with $12\ \Omega$ resistor.



Circuit Theorem

Now current I_0 will be

$$I_0 = \frac{36 \text{ V}}{(4 \Omega + 2 \Omega + 3 \Omega)} = \frac{36 \text{ V}}{9 \Omega}$$
$$= 4 \text{ A}$$

This 4A current will split between parallel resistors of 12Ω & 4Ω

So current I through 4Ω resistor is

$$I_{4\Omega} = I \frac{(3+1)\Omega}{(12+4)\Omega} = \frac{4 \times 12}{16} = 3 \text{ A}$$

$$I = 3 \text{ A}$$

Circuit Theorem

Now to verify reciprocity theorem we place $36V$ voltage source in series with 1Ω resistor and find current through 4Ω resistor. Redrawing the circuit as shown below,

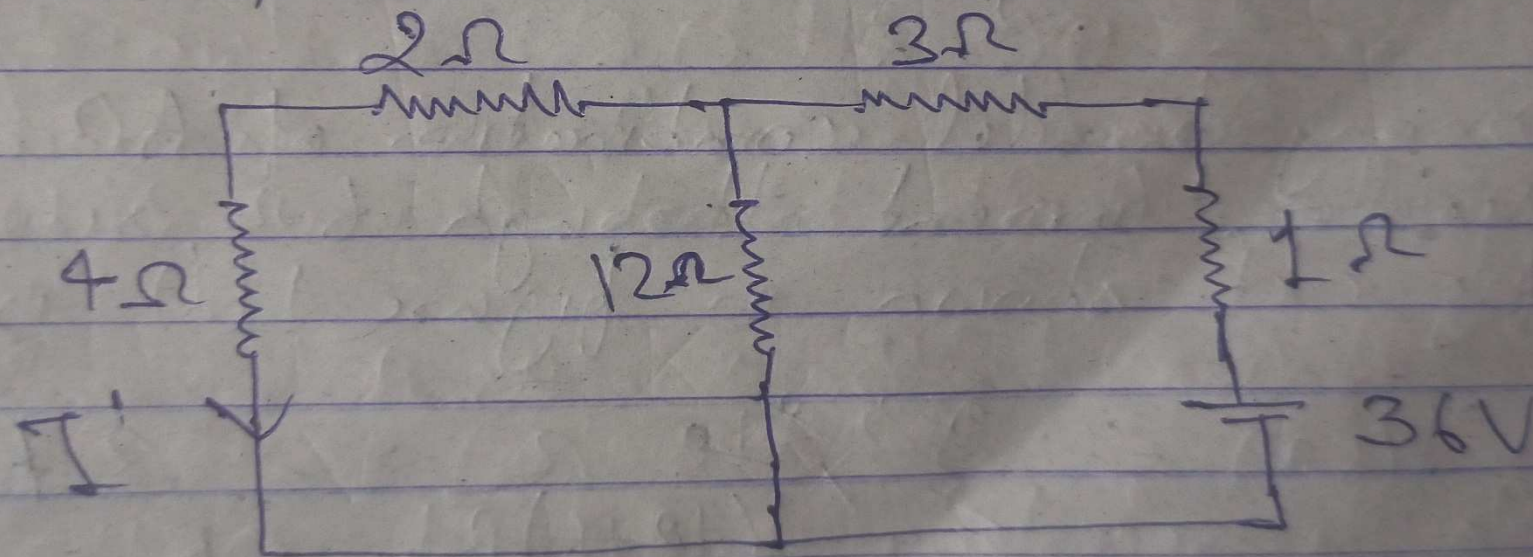
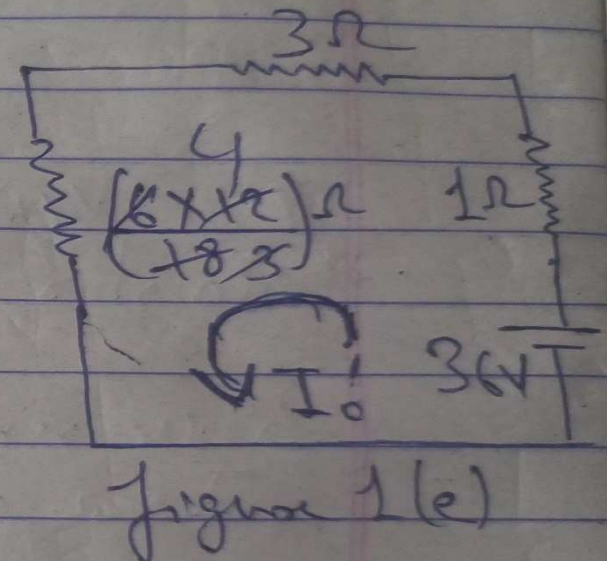
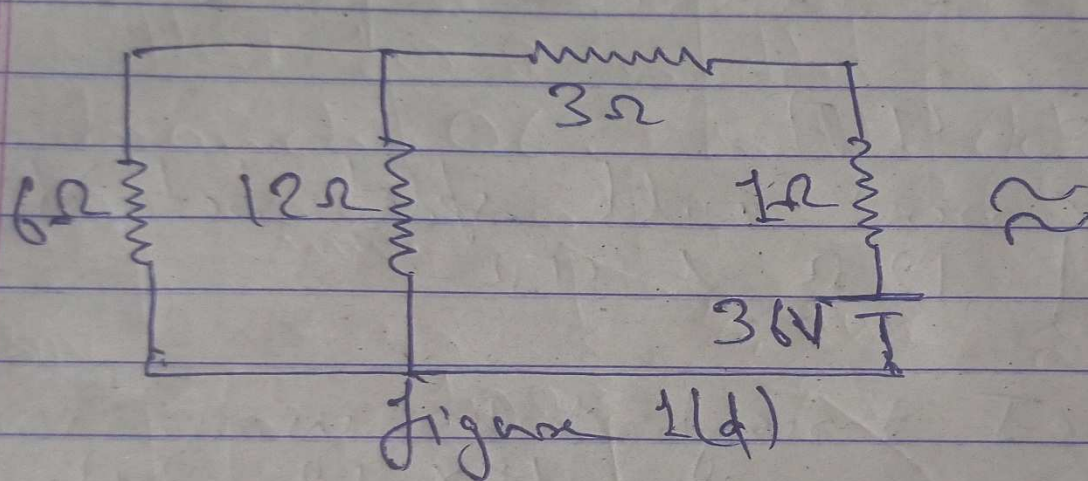


fig 1(c)

Circuit Theorem

In the ckt. shown in figure 1 (d) resistors are in series and then parallel to 12Ω resistor as shown below;



$$I_0' = \frac{36 \text{ V}}{(3\Omega + 1\Omega + 4\Omega)}$$

from ckt. of figure 1(e)

Circuit Theorem

from ckt. of figure 1(c)

$$I_0' = \frac{36 \text{ V}}{8 \Omega} = \frac{9}{2} \text{ A}$$

Now this $\frac{9}{2}$ A current will divide

among parallel combination of 6Ω & 12Ω resistor. Current through 6Ω resistor of ~~the~~ ckt. shown in figure 1(d) is

$$I(6\Omega) = \frac{9}{2} \times \frac{12\Omega}{(12+6)\Omega} = \frac{9}{2} \times \frac{12}{18} = 3 \text{ A}$$

Circuit Theorem

So current I' i.e. $I(4\Omega) = 3\text{ A}$.

This ~~pro~~ means that the ckt. follows reciprocity theorem or we can say that reciprocity theorem is applicable in given problem circuit.

$$\frac{I}{V} = \frac{I'}{V} = \frac{3}{36} = \frac{1}{12} \quad \text{Transfer Admittance}$$

Circuit Theorem

Q2) Show the validity of reciprocity theorem in figure ckt. of figure 2(a).

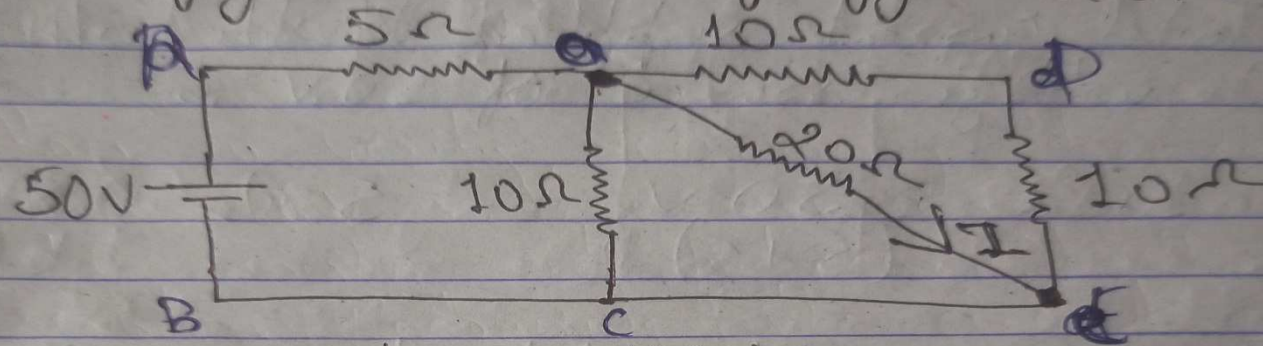


figure 2(a)

Solution \Rightarrow Let's redraw the ckt. by combining the parallel series resistance of $10\Omega + 10\Omega = 20\Omega$ and then will \parallel of 20Ω resistor as shown in figure 2(b)

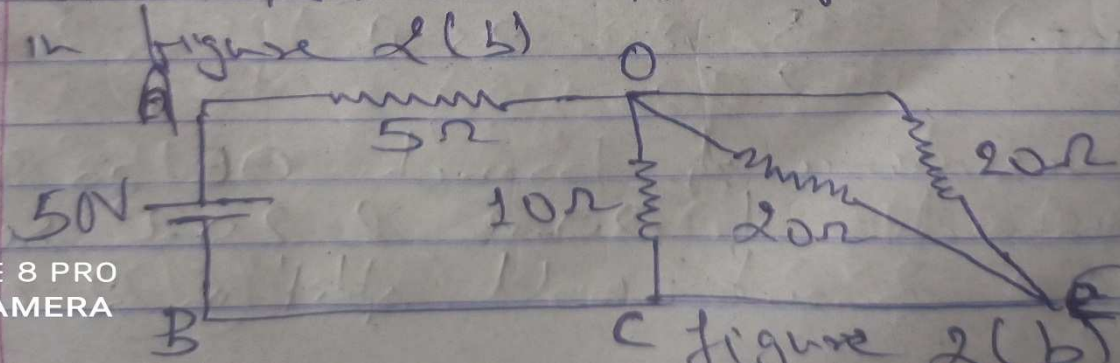
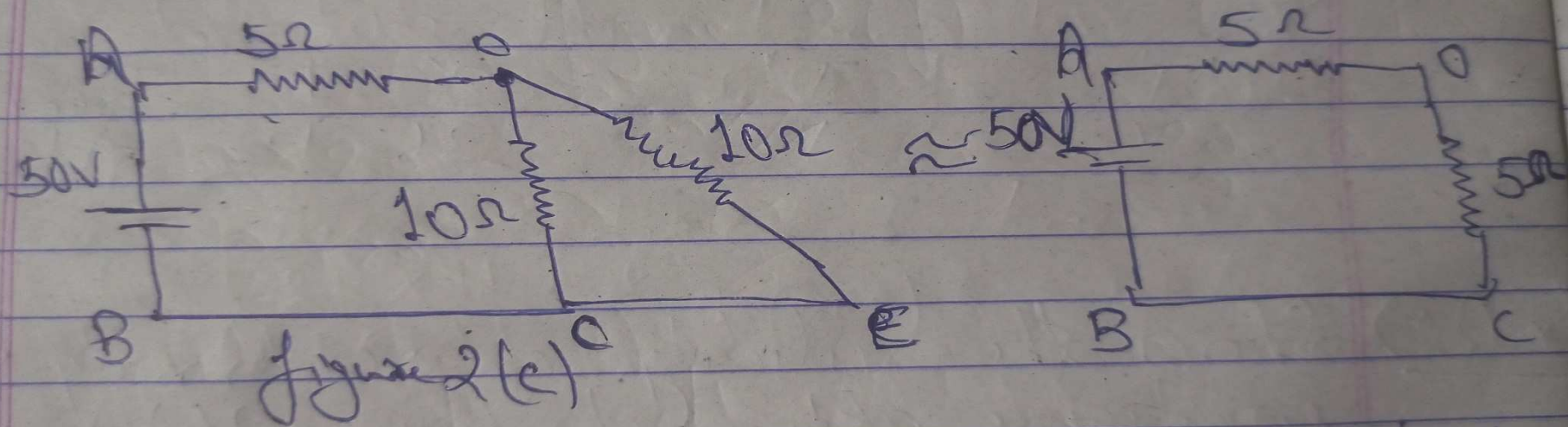


figure 2(b)

Circuit Theorem

Now 20Ω is in parallel with 20Ω resistors, so,
 $R_{eq}(20||20) = \frac{10 \times 20 \times 20}{(20+20)} = 10\Omega$

This 10Ω is in parallel to the 10Ω resistor of branch OC, so the ckt will be now,



Total current supplied to the n/w ~~due~~ by the voltage source of $50V$ is,

Circuit Theorem

$$I_{\text{total}} = \frac{50 \text{ V}}{(5+5) \Omega} = 5 \text{ A}$$

Thus 5 A will split at O between branch OC & OE in two equal parts because the value of resistance is equal in both branch.

$$I(10\Omega) = \frac{5 \text{ A}}{2} = 2.5 \text{ A}$$

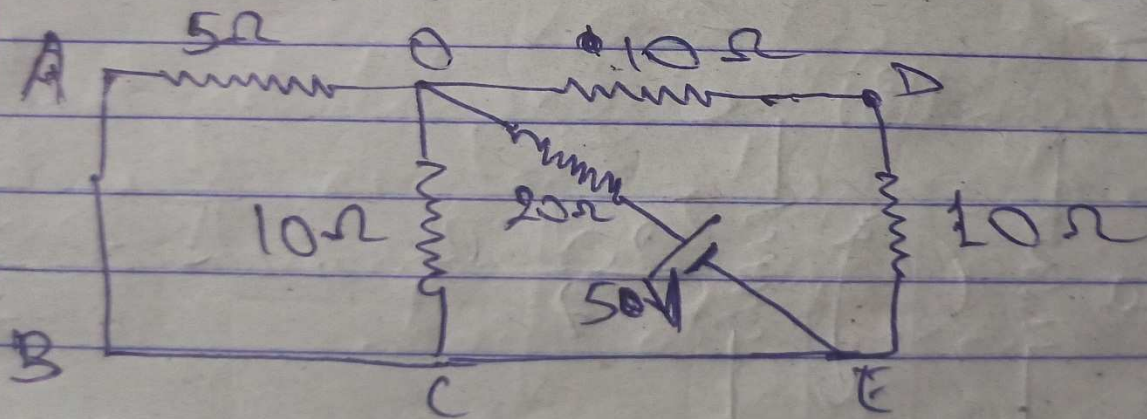
As the resistance of branch OE is due to 110 Ω combination of 20 Ω & 20 Ω resistors so again 2.5 A current will split in 2 equal parts

Circuit Theorem

$$I(20\Omega) = \frac{2.5 \text{ A}}{2} = 1.25 \text{ A}$$

So the current through branch OE is 1.25 A i.e. through 20Ω resistor connected between branch OE.

Now to verify reciprocity theorem, we connect the 50V voltage source in series with 20Ω resistor of branch OE and draw the ckt as shown in figure 2(d)



Circuit Theorem

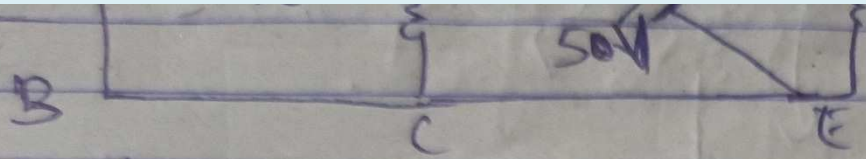


figure 2(d)

from ckt shown in figure 2(d) we can see that 5Ω resistor at branch OA and 10Ω resistor at branch OC are || so equivalent ckt can redrawn as below.

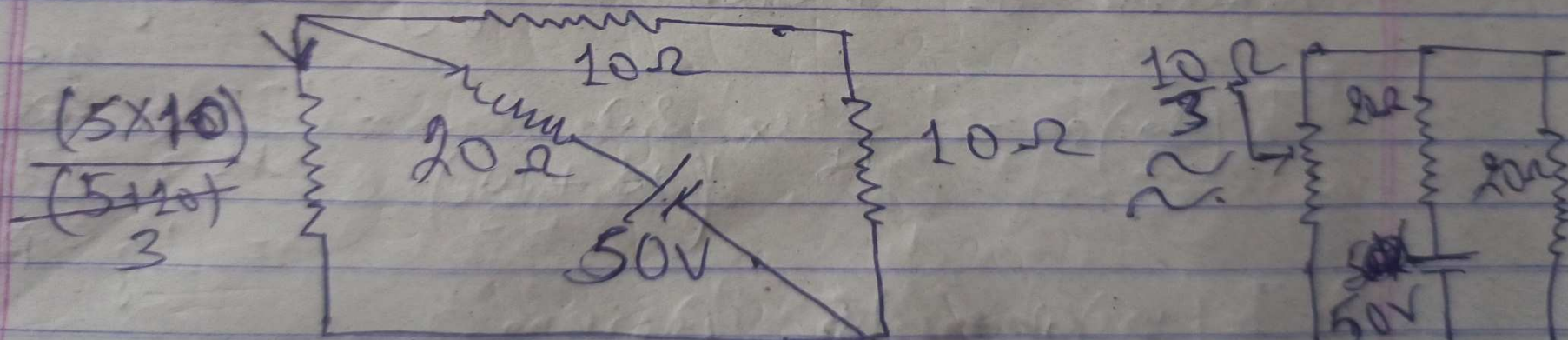
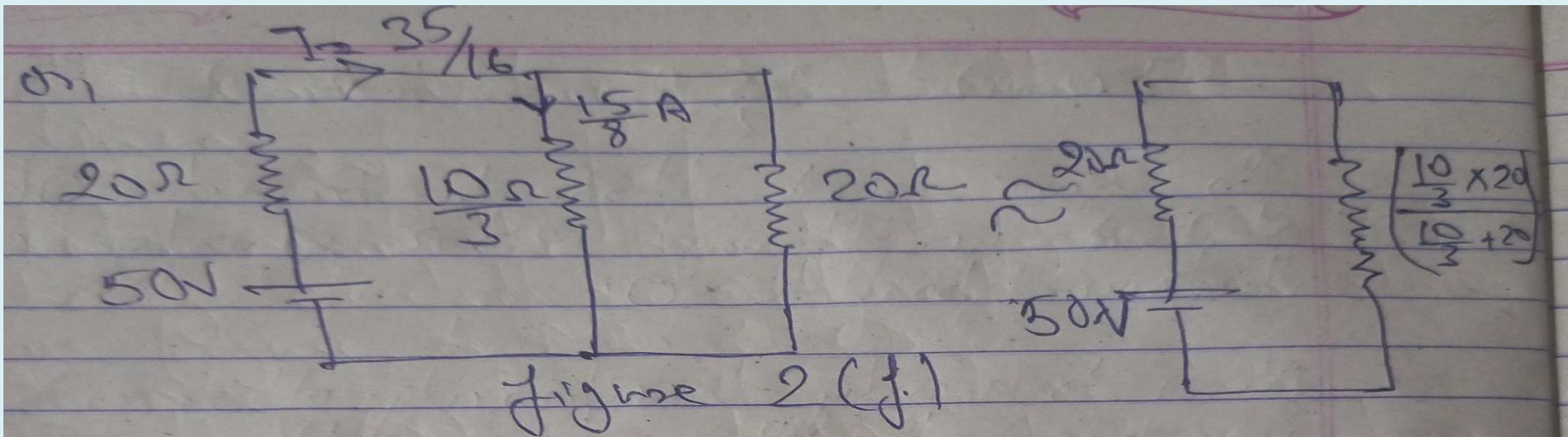


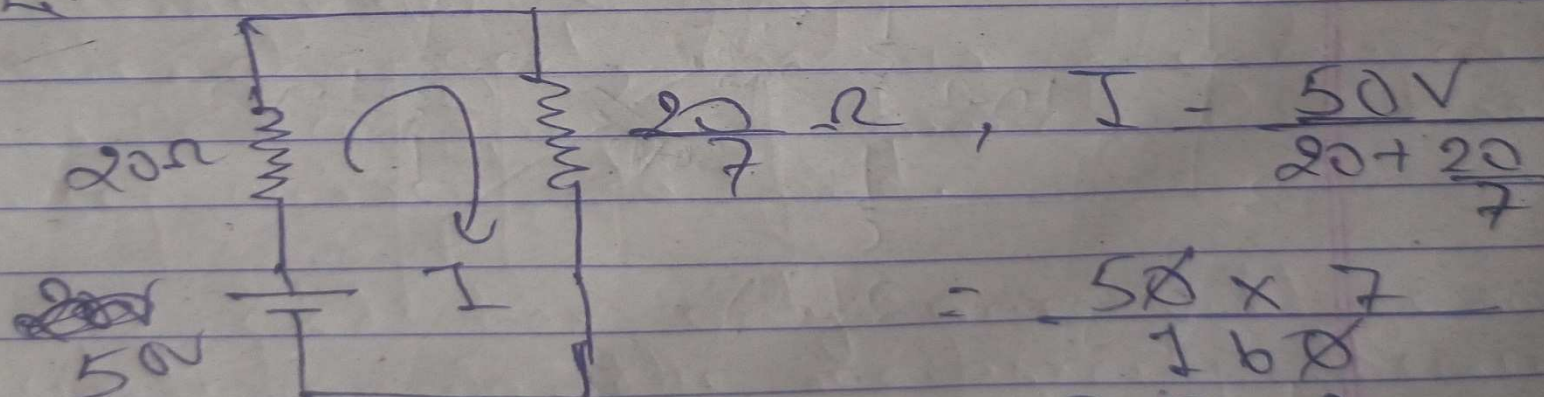
figure 2(e)

Circuit Theorem

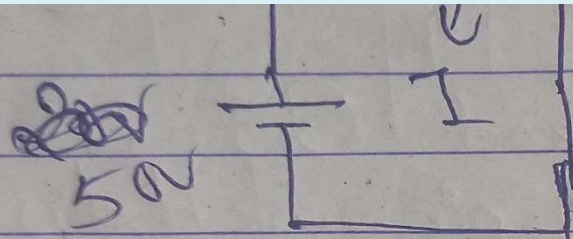


$$\frac{\frac{10 \times 20}{3}}{\frac{10}{3} + 20} = \frac{200/3}{70/3} = \frac{20}{7} \Omega$$

or the



Circuit Theorem


$$= \frac{50 \times 7}{16}$$
$$= \frac{35}{16} \text{ A}$$

This $\frac{35}{16}$ A will split among $\frac{10}{3} \Omega$ & 4Ω

resistors as shown in figure 2(f).

$$I \left(\frac{10}{3} \Omega \right) = \frac{35}{16} \times \frac{20}{\left(20 + \frac{10}{3} \right)} = \frac{35}{16} \times \frac{20 \times 3}{70}$$
$$= \frac{15}{8} \text{ A}$$

$$I \left(\frac{10}{3} \Omega \right) = \frac{15}{8} \text{ A}$$

Circuit Theorem

This $\frac{15}{8}$ A current will split among the 5Ω & 10Ω resistor of branch OA & OC respectively so, current through 5Ω resistor is

$$I(5\Omega) = \frac{15}{8} \times \frac{10}{(10+5)} = \frac{10}{8} \text{ A} = 1.25 \text{ A}$$

~~Which~~ This verifies the applicability of reciprocity theorem

Circuit Theorem

Q3) Use the data of figure (a). Find the value of current I in the circuit of the figure (b).

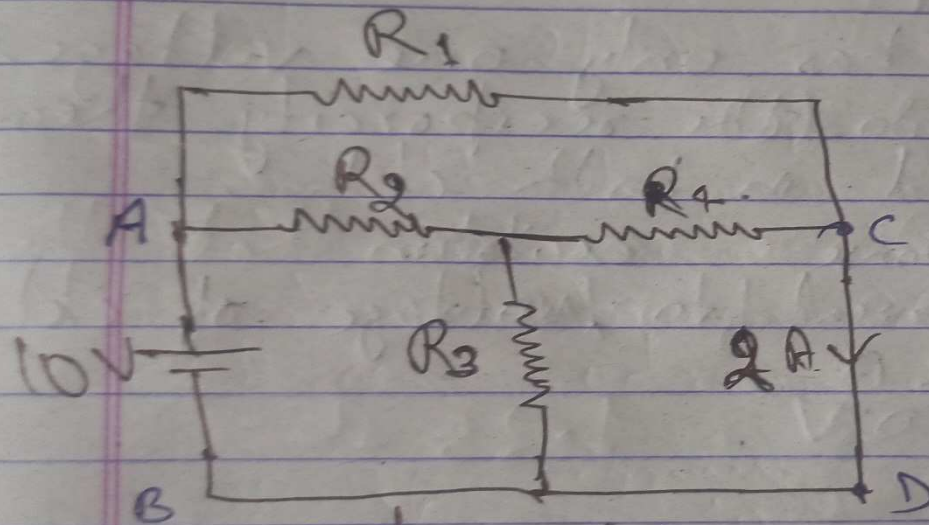


figure (a)

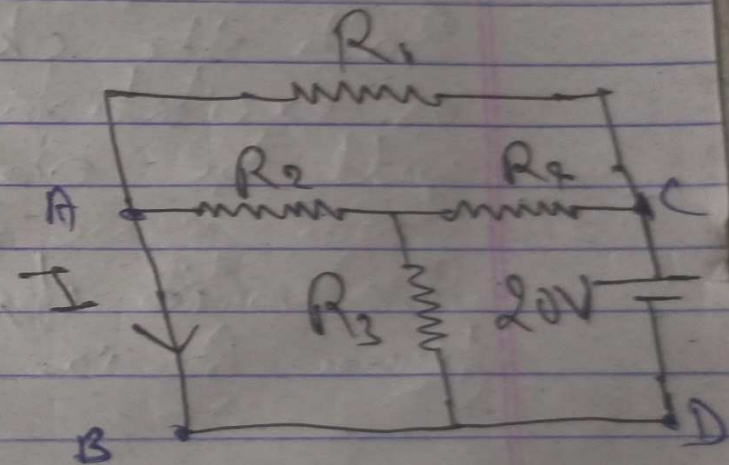


figure (b)

Soln:

As we can see that ckt of figure (a) & (b) are ~~same~~ having same connection of all resistor and having

Circuit Theorem

the same value, so both ckt is linear,

Let's name the branch of figure (a) & (b) having excitation and response as shown in figure as AB & CD

In figure (A) branch AB has excitation voltage while branch CD has response current ~~I~~ 1A,

While in figure (B) the excitation and response are inter changed, that is the branch CD has excitation and branch AB has response current to find.

Circuit Theorem

Both the ckt has only one voltage source as excitation, so the reciprocity theorem will be obeyed in both ckt.

∴ ratio of ^{response} current to ^{excitation} voltage of ckt of figure A is $\frac{2A}{10V}$

Now ratio of response current to excitation voltage of ckt of figure B is $\frac{I}{20V}$

As per reciprocity theorem,

$$\frac{I}{20V} = \frac{2A}{10V} \Rightarrow I = \underline{4A} \text{ ans}$$

Circuit Theorem

Q4) for the n/w shown in below figure find the current I .

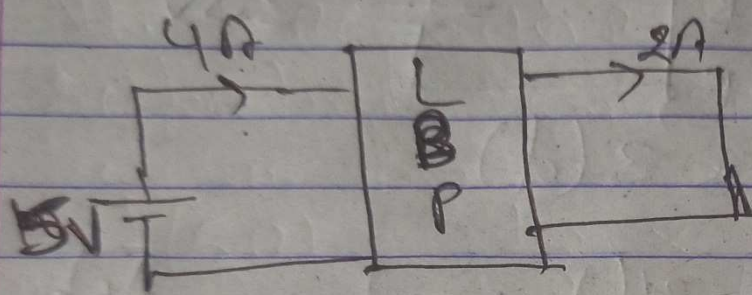


figure 3(a)

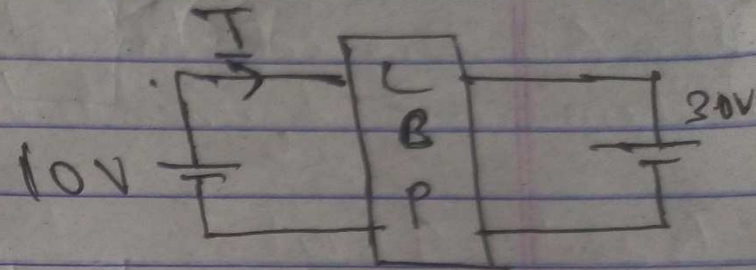


figure 3(b)

Ans: We have to find current I in figure 3(b).

As the n/w of figure 3(a) & 3(b) are LBP . i.e. Linear, Bilateral & Passive so the reciprocity theorem is applicable.

Circuit Theorem

In figure 3(a) the excitation voltage is 5V and response current is 2A, then according to reciprocity theorem if the excitation & response is changed then

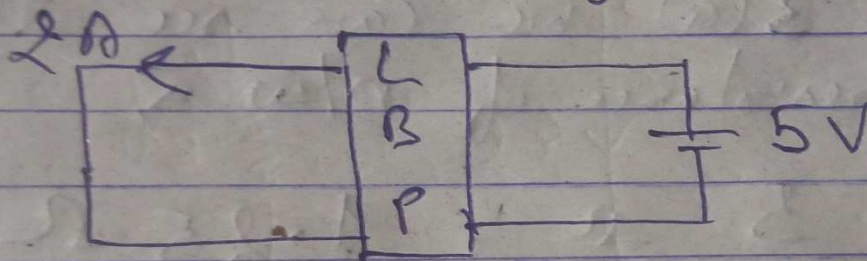


Figure 3(c)

When we scale 5V by 6 then response current will also be $2 \times 6 = 12A$.

So the ~~sk n/w~~ of figure 3(b) can be looked

Circuit Theorem

like,

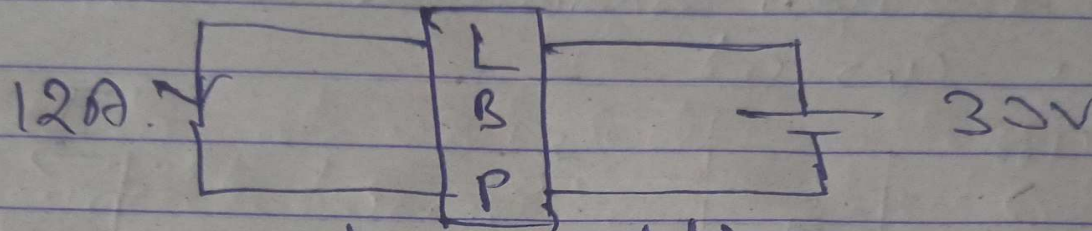
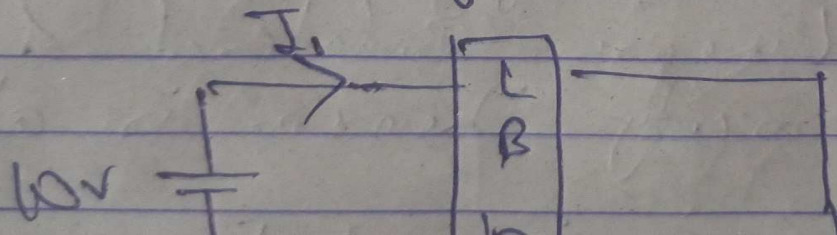


figure 3(d)

In n/w of figure 3(d) we have taken response due to only source that is 30V as per superposition theorem.

Now we will left 30V source and take 12A source find the current due to this,



Circuit Theorem

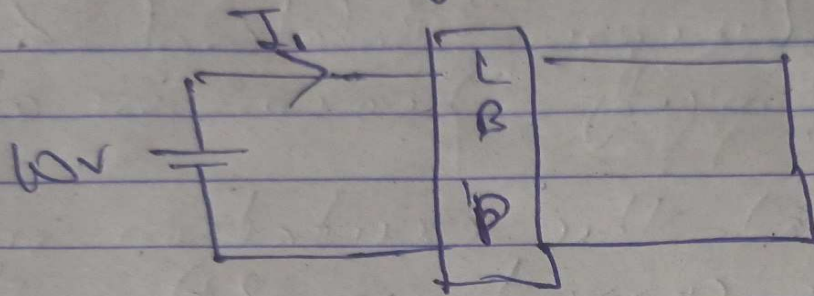


figure 3(e)

from figure 3(a) we can see that the internal resistance of n/w is,

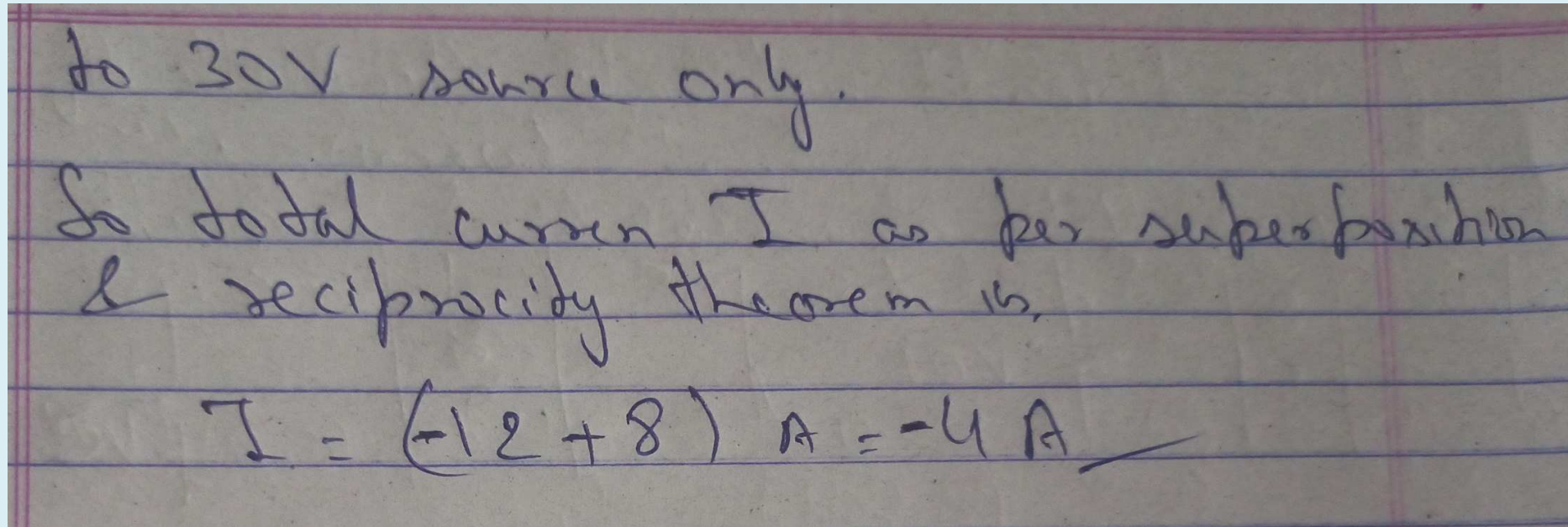
$$R_{in} = \frac{5}{4} \Omega = 1.25 \Omega$$

∴ current I_1 due to 10V source is.

$$I_1 = \frac{10V}{1.25} = \frac{1000^8}{125} A = 8A,$$

The 8A is in opposite to 12A current due

Circuit Theorem



In next lecture we will discuss Star-Delta Transformation. For any query or problem discuss on What's App or contact on number available.

THANK YOU