Paper 1, TDC Part-1
Chapter– 4, Circuit Theorems
Lecture - 9

By:

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Circuit Theorem

In this lecture we will discuss about 'Reciprocity' Theorem.

Like other theorems, this theorem is also very useful for those network which has independent source, is made of resistive element and the network is linear or bilateral.

So If we wish to apply reciprocity theorem in any network then following condition must be satisfied-

a) The network consist of independent source.
b) The network is made up of resistive elements only, that means n/w should be time invariant.
c) The network is linear and bilateral.

Circuit Theorem

It means that reciprocity theorem is not applicable:-

1) Network with of dependent source.

2) In time variant network.

3) In a non linear network.

A network or circuit in which reciprocity theorem is applicable is called reciprocal network/ circuit.

Circuit Theorem

Explanation of Reciprocity Theorem

Let us consider a passive n/w of linear elements.

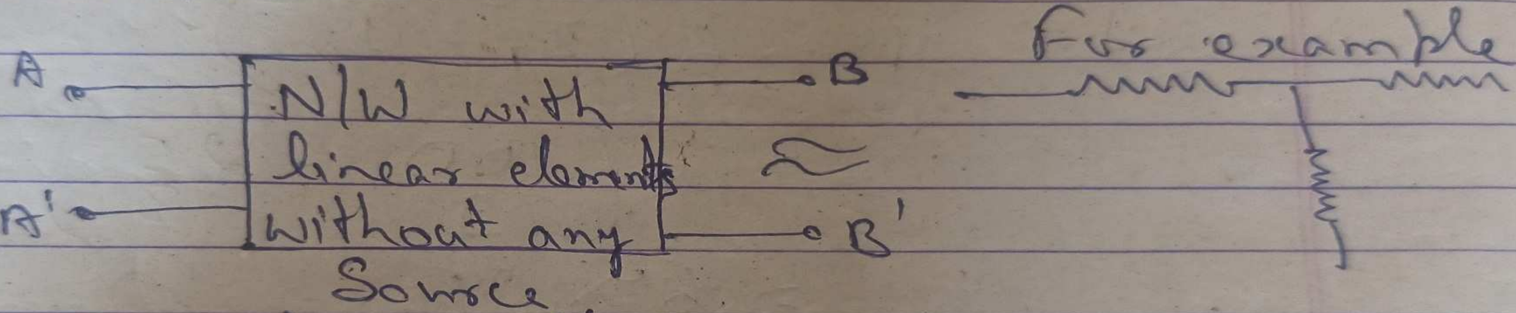

Figure 1(a)

Let the above n/w be excited by a voltage ~~vol~~ source which is independent at terminal AA'. Due to this voltage source the current I flows through the short circuit terminal BB' as shown in figure 1(b)

Circuit Theorem

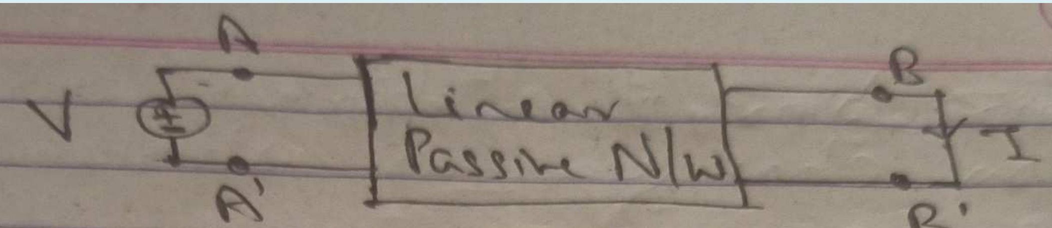

figure 1 (b)

Here 'V' is the supply voltage and 'I' is the output current, are mutually transferable and the ratio of V and I is called the transfer resistance.

Now as per the Reciprocity theorem if this supply voltage 'V' is connected between terminal BB', then the current response in the short circuited terminal AA' will be 'I' as shown in figure 1 (c) below

Circuit Theorem

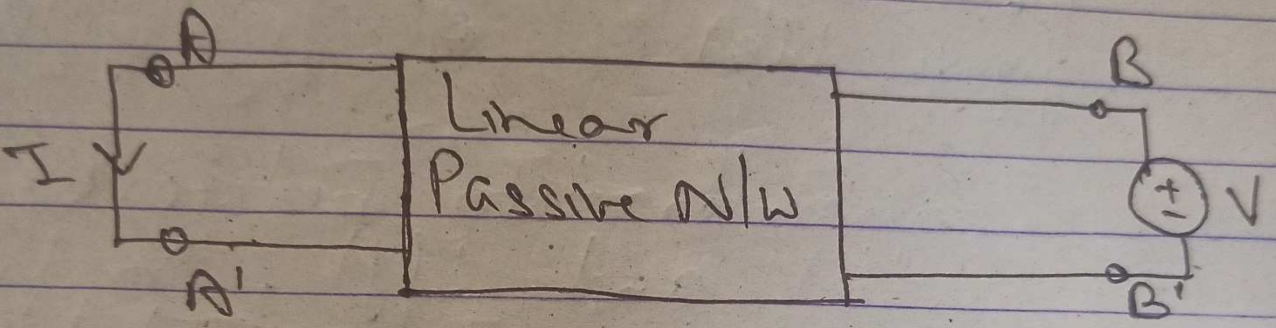

figure 1(c)

Here we should also note that, if the direction of supplying voltage ~~should~~ is ~~be~~ same then the direction of response current will be also same.

If the direction of voltage is change the

Circuit Theorem

the direction of response current will be also changed as shown in figure 1(d)

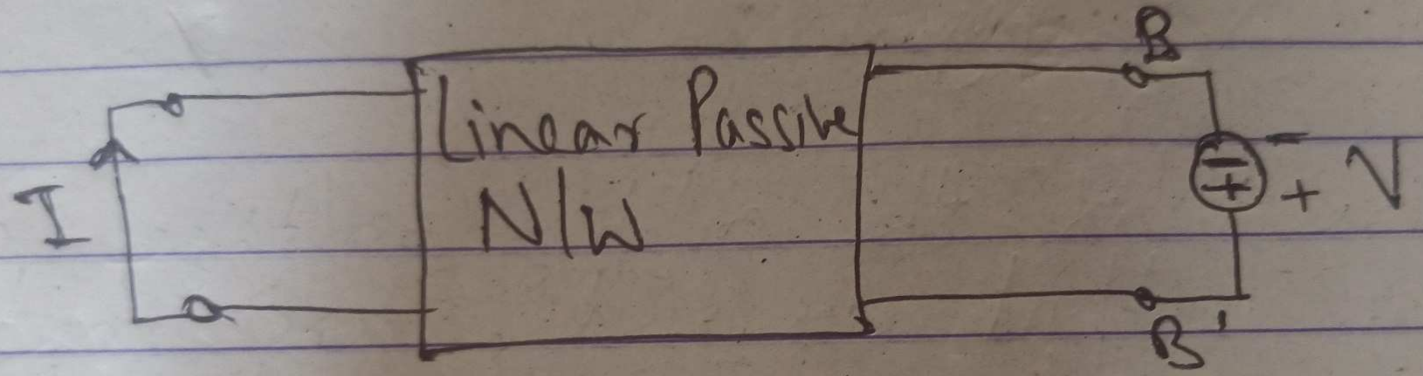

figure 1(d)

Proof of Reciprocity Theorem :-

Circuit Theorem

Proof of Reciprocity Theorem :-

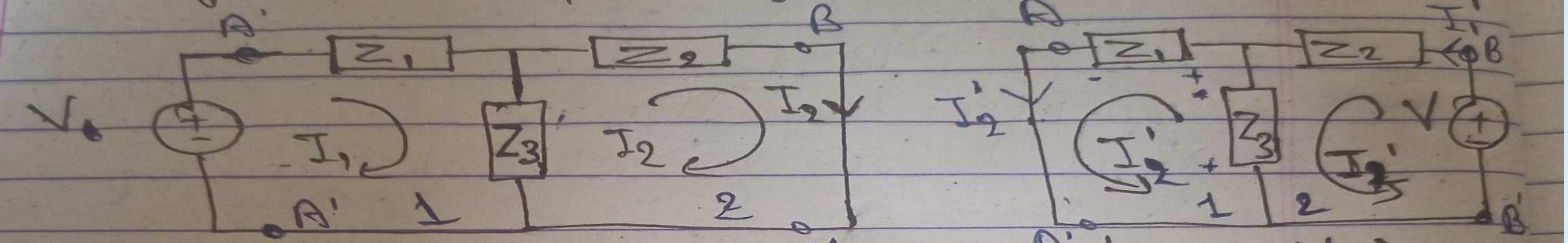

figure 1(e) figure 1(j)

Let us consider two network as shown in figure 1(e) and 1(j). In figure 1(e) the voltage source is connected between terminal AA' and response current is I_2 flowing in short circuit terminal BB'.

While in figure 1(j) the n/w is just as in the figure 1(e), but here the voltage source is connected between terminal BB' and response current I_2' is between short circuit terminal AA'.

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So As per reciprocity theorem,

$$\frac{I_2}{V} = \frac{I_2'}{V} \quad \text{or,} \quad \frac{V}{I_2} = \frac{V}{I_2'}$$

This is possible only when $I_2' = I_2$

Let us find the current I_2 in terminal BB' of figure 1(e)

Applying KVL in loop 1,

$$I_1 Z_1 + I_1 Z_3 - I_2 Z_3 = V$$

$$\text{or, } I_1 (Z_1 + Z_3) - I_2 Z_3 = V \quad \text{——— (i)}$$

Circuit Theorem

or, $I_1(Z_1+Z_3) - I_2 Z_3 = V$ ——— (i)

Now applying KVL in loop 2,

$-I_1 Z_3 + I_2 Z_2 + I_2 Z_3 = 0$

or,

$$I_1 Z_3 = I_2(Z_2+Z_3)$$

$$I_1 = \frac{I_2(Z_2+Z_3)}{Z_3}$$

putting value of I_1 in eqn. (i) we get,

$$\frac{I_2(Z_2+Z_3)(Z_1+Z_3)}{Z_3} - I_2 Z_3 = V$$

Circuit Theorem

or, $I_2(Z_1Z_2 + Z_1Z_3 + Z_2Z_3 + Z_3^2) - I_2Z_3^2 = VZ_3$

or, $$I_2 = \frac{VZ_3}{Z_1Z_2 + Z_3Z_1 + Z_2Z_3}$$

Now we will find I_2' for circuit shown in figure 1(f).

Applying KVL in loop1 of figure 1(f)

$$I_2'(Z_1+Z_3) - I_1'Z_3 = 0$$

$$I_1' = \frac{I_2'(Z_1+Z_3)}{Z_3}$$

Circuit Theorem

KVL in loop 1 of figure 1(f)

$$I_1'(Z_2 + Z_3) - I_2' Z_3 = V$$

putting value of I_1' we get.

$$\frac{I_2'(Z_2+Z_3)(Z_1+Z_3)}{Z_3} - I_2' Z_3 = V$$

$$I_2'(Z_1 Z_2 + Z_3 Z_1 + Z_2 Z_3 + Z_3^2) - I_2' Z_3^2 = V Z_3$$

Or, $$I_2' = \frac{V Z_3}{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}$$

Circuit Theorem

As, the value of I_2 and I_2' are same so the ratio of response to excitation for both n/w of figure 1(e) & 1(f) are same i.e.

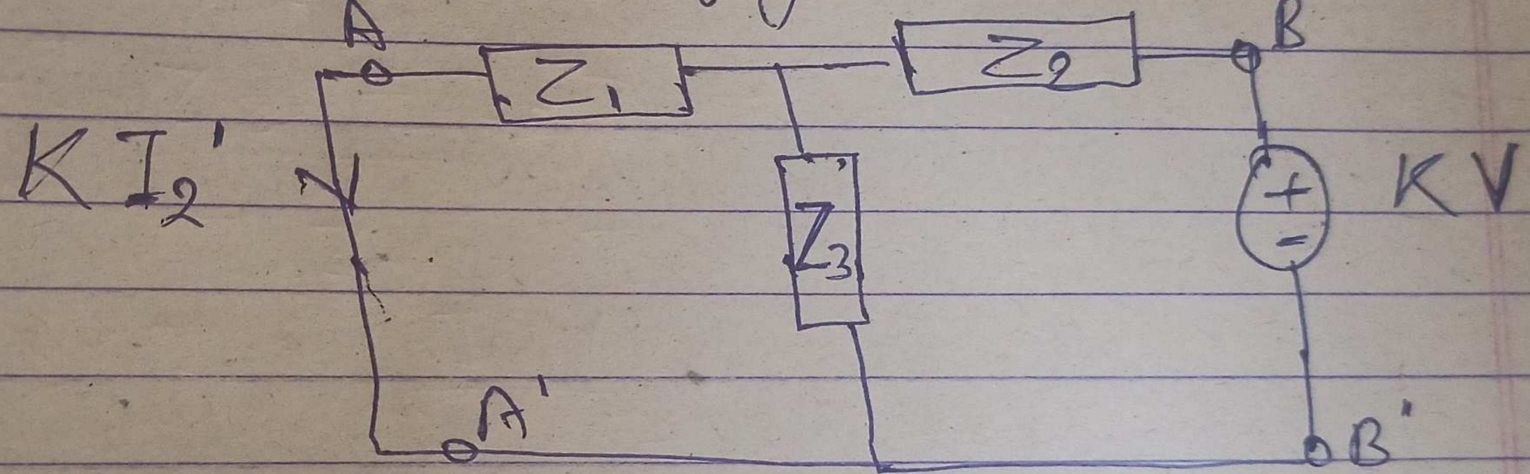
$$\frac{I_2}{V} = \frac{I_2'}{V} \quad \text{or,} \quad \frac{V}{I_2} = \frac{V}{I_2'}$$

This proves the reciprocity theorem.

Similarly if the value of voltage source is multiplied with any number then the response current will be also multiplied with the same number i.e

Circuit Theorem

E.g. for network of figure 1(f),



So, $$\frac{I_2}{V} = \frac{KI_2'}{KV} = \frac{I_2'}{V}$$

This proves the reciprocity theorem

Circuit Theorem

In next lecture we will discuss problem based on reciprocity theorem.

THANK YOU