# Debye shielding / Debye length

One of the most important properties of a plasma is the shielding of every charge in the plasma by a cloud of oppositely charged particles called Debye shielding. The distance over which the influence of the electric field of an individual charged particle is felt by the other charged particles inside the plasma is called Debye length.

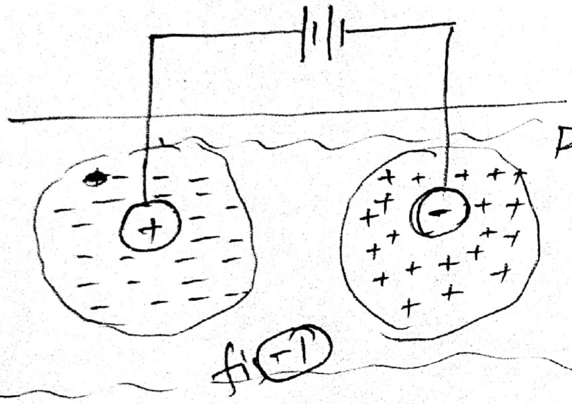

fig-1

To make it clear, let us try to put an electric field inside a plasma by inserting two charged balls connected to a battery (fig-1). The balls would attract particles of opposite charge, and almost immediately a cloud of ions would surround the -ve ball and a cloud of electrons would surround the +ve ball. (we assume that a layer of dielectric keeps the plasma from actually recombining on the surface or the battery is large enough to maintain the potential in spite of this). If the plasma were cold no thermal motions, there would be just as many charges in the cloud as in the ball, the shielding would be perfect and no electric field would be present in the body of the plasma outside the clouds. On the other hand, if the temp is finite, those particles that are at the edge of the cloud, where the electric field is weak have enough thermal energy to escape from the electrostatic potential well. The edge of the cloud then occurs at the radius where the potential energy is approximately equal to the thermal energy of the particles. This radius is called Debye Shielding distance or the Debye-Huckel radius or the simply Debye length.

## Derivation of Debye Length

Let us take some test charge $+Q$ within the plasma at any instant and choose a spherical co-ordinate system whose origin coincides with the position of the test charge $+Q$. The electrostatic potential $\phi(r)$ is established near the test charge, due to the combined effects of the test charge and distribution of charged particles surrounding it.

The number density of the electrons is given by Boltzmann Law

$$n_e(r) = n_0 \exp\left[\frac{e\phi(r)}{KT}\right] \quad \text{— (I)}$$

and the number density of the ions is given by

$$n_i(r) = n_0 \exp\left[\frac{-e\phi(r)}{KT}\right] \quad \text{— (II)}$$

where $K$ is the Boltzmann Const and $T$ is the temp in Kelvin

At large distances $n_e(\infty) = n_i(\infty) = n_0$ is the number densities = Constant

By Poisson's Equation

$$\nabla^2\phi(r) = -4\pi \times \text{excess charge}$$

$$= -4\pi(n_i e - n_e e)$$

$$= -4\pi e(n_i - n_e) \quad \text{— (III)}$$

In spherical Polar co-ordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi(r)}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi(r)}{\partial\theta}\right)$$
$$+\frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2\phi(r)}{\partial\phi^2}\right)=-4\pi e(n_i-n_e) \quad \text{— (IV)}$$

[ In Case of steady state Problem under Conservative electric field

$$\vec{E}(r) = -\nabla\phi(r) \quad [\text{-ve Potential gradient}]$$

Its divergence is

$$\vec{\nabla}.\vec{E} = \vec{\nabla}.-\vec{\nabla}\phi(r) = -\nabla^2\phi(r)$$

By Gauss's law $\vec{\nabla}.\vec{E} = \rho/\epsilon_0$ where $\rho$ = charge density $= \frac{q}{V} = q/\frac{4}{3}\pi r^3$

Therefore $\nabla^2\phi(r) = -\rho/\epsilon_0$ which is Poisson's eqn

spherical Polar Co-ordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}-\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}-\right)$$
$$+\frac{1}{r^2\sin^2\phi}\frac{\partial^2}{\partial\phi^2}- \quad ]$$

If the charge distribution is symmetric then

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi(r)}{\partial r}\right) = -4\pi e(n_i-n_e) \quad \text{— (V)}$$

Substituting the value of eqn (I) & (II) in eqn (V) we get the Poisson's equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi(r)}{\partial r}\right) = -4\pi e\left[n_0\left\{\exp^{-}\left(\frac{e\phi(r)}{KT}\right) - \exp\left(\frac{e\phi(r)}{KT}\right)\right\}\right]$$

$$\left[\begin{array}{l} \because \sinh x = \dfrac{e^{x}-e^{-x}}{2} \\ \text{So } e^{x}-e^{-x} = 2\sinh x \end{array}\right]$$

$$= 4\pi n_0 e\left[\exp\left(\frac{e\phi(r)}{KT}\right) - \exp\left(\frac{-e\phi(r)}{KT}\right)\right]$$

$$= 8\pi e n_0 \sinh\left(\frac{e\phi(r)}{KT}\right) \quad \text{--- (VI)}$$

If $KT \gg e\phi(r)$

then $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi(r)}{\partial r}\right) = \cancel{8\pi e^2 n_0 \sinh(}$$
$$= 8\pi e^2 n_0 \frac{\phi(r)}{KT}$$

or $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi(r)}{\partial r}\right) = 8\pi e^2 n_0 \frac{\phi(r)}{KT} \quad \text{--- (VII)} = \nabla^2\phi$$

This differential equation has an infinite no. of solutions, one of which suit us that satisfies the boundary condition

At $r \to 0$, $\phi(r) = \frac{Q}{r}$

at $r \to \infty$, $\phi(r) = 0$

Then the solution is

$$\phi(r) = \frac{Q}{r}e^{-\frac{r}{\lambda_D}}$$

where $\lambda_D = \sqrt{\dfrac{KT}{4\pi n_0 e^2}}$

This is so called Debye Hückel radius or Debye shielding distance or Debye length

* Thus when $r = \lambda_D$, the effect of charge $Q$ becomes $\frac{1}{e}$th of its original value. It therefore follows from the above eqn that if the linear dimension of the region occupied by an ionised gas of given electron concentration is much greater than $\lambda_D$, then inside the region $n_e \neq n_i$ and under this condition a considerable departure of $n_e$ from $n_i$ will give rise to an electric field which will eject particles of opposite sign. The mechanism will automatically maintain the neutrality of $n_e$ & $n_i$