Analytical Geometry of two dimensions

Parabola

Q.) To find the equation of parabola whose locus is the point (h,k) and directrix is the line $x+y=0$. Find the standard equation of parabola.

Sol:- Let (h,k) be the locus and $P(x,y)$ be any locus and $P(x,y)$ be any point on the directrix whose equation is $x+y=0$

By definition

$$\frac{SP}{PM} = 1$$

$$\Rightarrow SP = PM$$

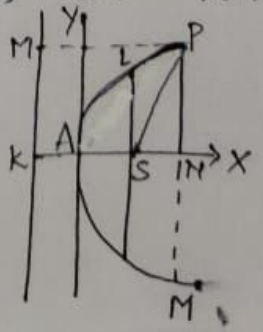
$$\Rightarrow SP^2 = PM^2$$

$$\Rightarrow (x-h)^2 + (y-k)^2 = \frac{(x+y+y)^2}{\alpha^2+\beta^2}$$

This is the required Equation of parabola.

Standard Equation of Parabola:-

Let S be the focus and KM the directix. We draw



let $SK \perp KM$ from S.

Bisect SK at A.

let $AK = AS = a$.

Now clearly, the coordinates of S are $(a,0)$ and the equation of the directrix is $x+a=0$

Let $P(x,y)$ be any point on the parabola. We join S to P and draw $PM \perp MK$ from $P(x,y)$

$\therefore PM = y, \quad AM = x$

$\therefore PM = KM = KA + AM = a + x$

Now, by definition of parabola $\frac{SM}{PM} = 1$

$\Rightarrow SP = PM$

$\Rightarrow SP^2 = PM^2$

$\Rightarrow (x-a)^2 + (y-0)^2 = (x+a)^2$

$\Rightarrow x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$

$\Rightarrow y^2 = 2ax + 2ax$

$\Rightarrow y^2 = 4ax$