Analytical Geometry of two dimensions

Problems :-

Q.) Find the equations of circles which passes through the origin and cuts the two circles $x^2+y^2-4x+6y+10=0$ and $x^2+y^2+12y+6=0$ orthogonally.

Sol:- Let the equation of the circle be

$$x^2+y^2+2gx+2fy+c=0 \quad \text{---(i)}$$

Since it passes through the origin, we have $c=0$

(i) becomes $x^2+y^2+2gx+2fy=0$ ---(ii)

Since (ii) cuts the circle $x^2+y^2-4x+6y+10=0$ orthogonally.

so, $-4g+6f=0+10$

i.e. $-4g+6f-10=0$

$\Rightarrow -(4g-6f+10)=0$

$\Rightarrow 4g-6f+10=0$

Again since (ii) cuts the circle $x^2+y^2+12y+6=0$

so, $12f-6=0$

$\Rightarrow 6(2f-1)=0$

$\Rightarrow 2f-1=0$

Thus we have $4g-6f+10=0$ ---(iii)

and $2f-1=0$ ---(iv)

From (iv) $f = 1/2$

putting the value of f in (iii) we get

$4g - 3 + 10 = 0 \Rightarrow 4g + 7 = 0$

$\Rightarrow g = \frac{7}{4}$

putting the value of f and g in equation (ii) we get

$x^2+y^2 + 2(-7/4)x^2 + (1/2)y = 0$

$\Rightarrow x^2+y^2 - \frac{7x^2}{2} + y = 0$

which is the required equation of circle.

Q(1) Find the Co-ordinates of the radical centre of the three circles $x^2+y^2 = 9$, $x^2+y^2-2x-2y = 5$ and $x^2+y^2+4x+6y = 19$

sol:- Radical axis of first and second circles is

$-2x - 2y - 5 + 9 = 0$

or, $x + y = 2$ ——— (i)

Radical axis of first and third circles

$4x + 6y - 19 + 9 = 0$

$\Rightarrow 4x + 6y - 10 = 0$

$\Rightarrow 2x + 3y = 5$ ——— (ii)

Solving (i) and (ii) we get the radical centre as (1, 1)