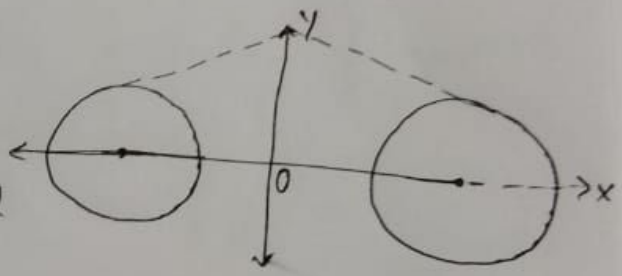
ANALYTICAL GEOMETRY OF TWO DIMENSIONS

Co-axial circles :-

Q) Define Coaxial system of circles. Find the equation of Co-axial system of circles and discuss their limiting points.

Definition :- A system of circles is said to constitute a Co-axial system of circles if the radical axis of every pair of circles of the system is the same straight line. We choose the line of centres as x-axis and the common radical axis as y-axis.



Hence the centre of every circle of the Co-axial system will be $(k, 0)$.

$\therefore$ The equation of the two circles are

$$x^2 + y^2 + 2g_1 x + c_1 = 0 \quad \text{—— (i)}$$

and $$x^2 + y^2 + 2g_2 x + c_2 = 0 \quad \text{—— (ii)}$$

The radical axis of the circles (i) and (ii) is

$$x^2 + y^2 + 2g_1 x + c = x^2 + y^2 + 2g_2 x + c_2$$

$$2(g_1 - g_2)x + c_1 - c_2 = 0$$

But we have taken the radical axis as y-axis $\therefore x = 0$.

$$\Rightarrow c_1 - c_2 = 0 \Rightarrow c_1 = c_2 = c \text{ (say)}$$

The equation of the circles (i) and (ii) becomes $x^2+y^2+2g_1x+c=0$ and $x^2+y^2+2g_2x+c=0$

Thus the equation $x^2+y^2+2gx+c=0$ represents co-axial circle for different values of g.

Limiting points of co-axial system of circles

The equation of co-axial system of circles is

$$x^2+y^2+2gx+c=0 \quad \text{---(i)}$$

Equation (i) represents a circle of radius $\sqrt{g^2-c}$ and centre $(-g, 0)$

If $\sqrt{g^2-c}=0$ i.e. if $g^2=c$

i.e. $g=\pm\sqrt{c}$.

The radius of the circle become zero and equation (i) gives two circles of radius zero and centre $(\pm\sqrt{c}, 0)$

The circles of zero radii are called points and are called limiting points of the co-axial system.