Cardinal Points of a Co-axial optical System (a Thick lens)

There are six cardinal points of a thick lens.

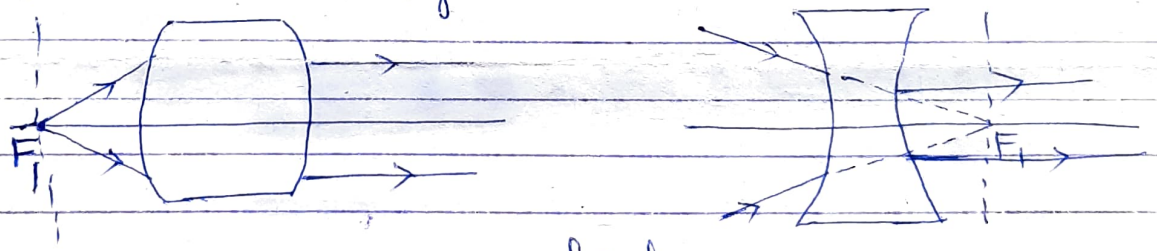
(1) Two focal points.
(2) Two principal points.
(3) Two nodal points.

These points are also known as "Gauss points" (1841)

(1) Focal points - The focal points are a pair of points lying on the principal axis of the optical system.

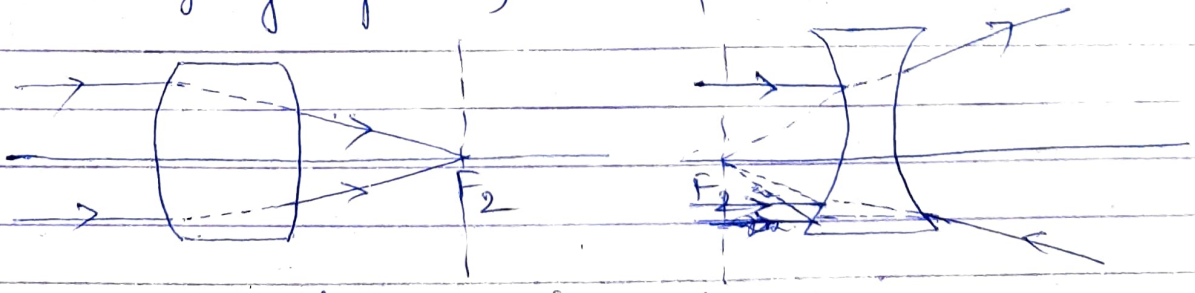
The first focal point is a point on the principal axis such that rays starting from (in converging system) or directed towards (in diverging system) this point, become parallel to the principal axis after refraction through the lens.

It may be defined as the object point on the principal axis for which the image point lies at infinity.



focal

The second point is a point on the principal axis such that a beam of incident rays parallel to the principal axis, after refraction converge to (in converging system) or appear to diverge from (in diverging system) this point.



It may be defined as the image point on the principal axis for which the object lies at infinity.

Principal Points

magnification = $\frac{\text{Size of the image}}{\text{Size of the object}}$.

Lateral or transverse (mm) = $\frac{h_1}{h_2}$ $\frac{v}{u}$

The principal points are a pair of conjugate points on the principal axis having unit positive linear transverse magnification. They are such that if one object be placed at one principal point, an erect image of the same size is formed at the other principal point.

Let us consider an incident ray 'a' parallel to the principal axis, after refraction through the lens passes through the second focal point F_2. The incident and emergent rays, when produced, intersects at A_2. The plane through A_2 and perpendicular to the axis is called the "Second principal plane" and its point of intersection with the axis H_2 is called the "Second principal point."

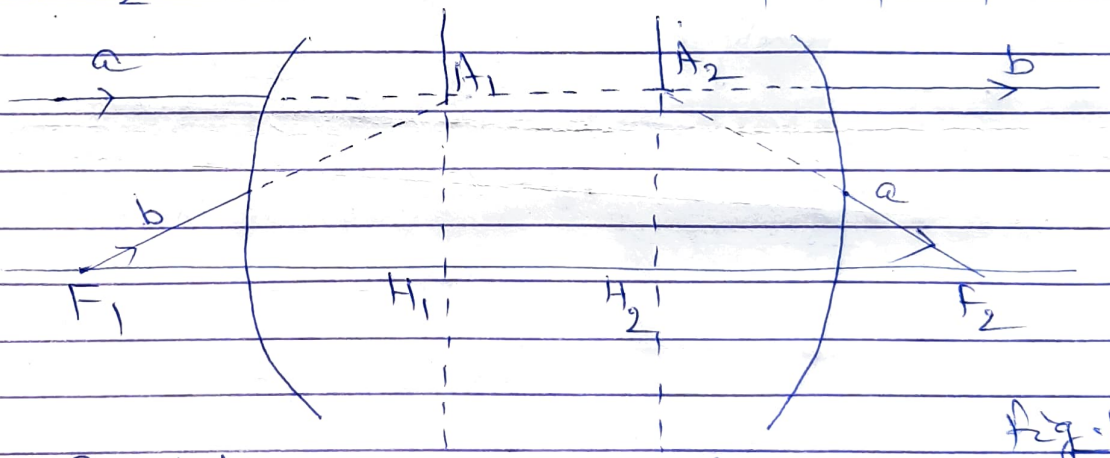


Fig. 1

Similarly, the incident ray 'b' passing through the first focal point F_1 become parallel to the principal axis after emerging through lens. The incident and emergent rays when produced, intersect at A_1. The plane through A_1 which is perpendicular to the principal axis is called the "first principal plane" and its point of intersection with the principal axis is called the "first principal point."

[The incident ray 'b' has been so chosen that the corresponding emergent ray lies at the same distance above the axis as the incident ray 'a'. It is thus seen that the incident rays a and b are converging towards the point F_1 and the corresponding emergent rays

appear to diverge from point A_2. Hence A_2 is the image of A_1 where $H_1A_1 = H_2A_2$. Hence A_1 and A_2 are conjugate points.

The distance H_1F_1 is called the "first focal length (f_1) and the distance H_2F_2 is called the "second focal length" (f_2) of the system.

(iii) Nodal points →

The nodal points are a pair of conjugate points on the principal axis of the system having unit positive angular magnification. They are such that an incident ray directed towards one nodal point emerges through the other nodal point in a direction parallel to its original direction.

Let H_1A_1 and H_2A_2 be the first and second principal planes of the optical system and OF_1 and OF_2 be its first and second focal planes respectively.

Let O be any point on the first focal plane and OA_1 is the ray of light parallel to the axis. The ~~conjugate~~ ray OA_1 will pass through second focal point F_2 after emerging from the point A_2 on the second principal plane such that

$$A_1H_1 = A_2H_2.$$

Consider another ray OP_1 parallel to A_2F_2 and intersecting the first principal plane at point P_1. This ray will emerge out from P_2 on the second principal plane such that $H_1P_1 = H_2P_2$ and will become parallel to A_2F_2, because both the rays start from O, a point on the first focal plane. The points of intersection of the incident ray OP_1 and the conjugate emergent ray P_2I with the principal axis AB are the two nodal points. It is obvious that N_1 and N_2 are a pair of conjugate points and the incident ray OP_1 is parallel to the corresponding emergent ray P_2I.

$$\frac{\tan(-\theta_1)}{\tan\theta_2} = \frac{\tan\theta_1}{\tan\theta_2} = 1. \quad \text{---------- (1)}$$

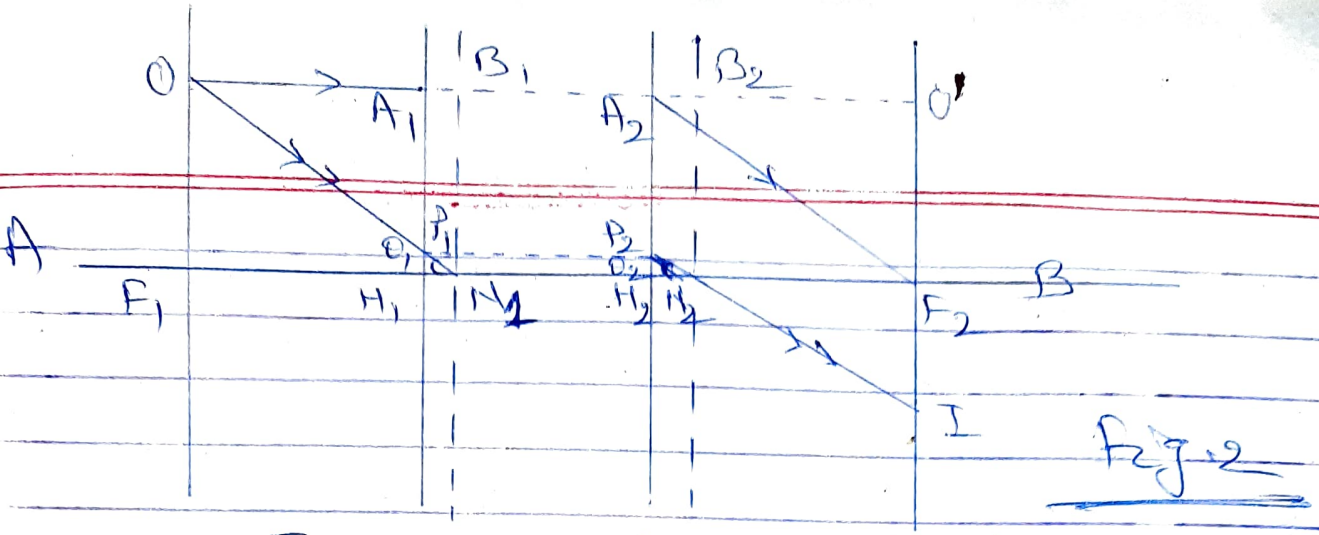


Fig-2

The planes passing through the nodal points N_1 and N_2 and normal to the principal axis are called the first and second nodal planes respectively.

In right angled triangles $P_1N_1H_1$ and $P_2N_2H_2$,

$P_1H_1 = P_2H_2$ and $\angle P_1N_1H_1 = \angle P_2N_2H_2$; $(\because \theta_1 = \theta_2)$

$\therefore$ triangles $P_1N_1H_1$ and $P_2N_2H_2$ are Coggruent.

$\therefore$ $H_1N_1 = H_2N_2$ ——— (2) समानुरूप

Adding N_1H_2 on both the sides, we get

$H_1N_1 + N_1H_2 = N_1H_2 + H_2N_2$

$H_1H_2 = N_1N_2$ — (3)

i.e. the distance between two nodal points is equal to the distance between two principal points.

—X—