

(xiv) Conjugate of a matrix: Let  $A = (a_{ij})_{m,n}$  be a matrix.

Let  $\bar{a}_{ij}$  denote the complex conjugate of  $a_{ij}$ . Then the matrix  $(\bar{a}_{ij})_{m,n}$  is called the conjugate of  $A$  and is denoted by  $\bar{A}$ .  
 Exompl:-  $A = \begin{bmatrix} 1+i & 1-i \\ 1+2i & 2-2i \end{bmatrix}$ ,  $\bar{A} = \begin{bmatrix} 1-i & 1+i \\ 1-2i & 2+2i \end{bmatrix}$

(xv) Transposal Conjugate of a matrix: The transpose of  $\bar{A}$  is called the transposed conjugate of  $A$  and is denoted by  $A^\theta$ .  
 Thus,  $A^\theta = (\bar{A})' = (\bar{A}')$

$$\begin{aligned} (A+B)^T &= A^T + B^T \\ (A+B)^\theta &= A^\theta + B^\theta \\ (AB)^\theta &= B^\theta A^\theta \\ (A^n)^\theta &= (A^\theta)^n \end{aligned}$$

(xvi) Hermitian matrix: A square matrix  $A$  is called a Hermitian matrix if  $A^\theta = A$ , i.e. if  $(i,j)^{\text{th}}$  element of  $A$  is the complex conjugate of the  $(j,i)^{\text{th}}$  element of  $A$ .

(xvii) skew-Hermitian matrix: A square matrix  $A$  is called skew Hermitian if  $A^\theta = -A$ .

(xviii) Unitary matrix: A square matrix

$A$  is called a unitary matrix if  $A^\theta A = A A^\theta = I$ .

Example:-  $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1+i & 1 \\ -1 & 1-i \end{bmatrix}$   
 Proof:-  $\bar{A} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1-i & 1 \\ -1 & 1+i \end{bmatrix}$

$$A^\theta = (\bar{A})^T = (\bar{A})' = \frac{1}{\sqrt{3}} \begin{bmatrix} 1-i & -1 \\ 1 & 1+i \end{bmatrix}$$

$$A A^\theta = \frac{1}{3} \begin{bmatrix} 1+i & 1 \\ -1 & 1-i \end{bmatrix} \begin{bmatrix} 1-i & -1 \\ 1 & 1+i \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1+1+1 & 0 \\ 1+1-i+i & 1+1+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$