Estimation and Confidence Intervals

Part 4

Point and Interval Estimates for Mean – Population Standard deviation (σ) known - Examples

- Confidence Interval for Population Mean with Population Standard deviation (σ) known $= \bar{X} \pm z\frac{\sigma}{\sqrt{n}}$

Where,

$\bar{X}$ = Sample Mean

z = z-value for particular confidence level

σ = Population Standard deviation

n = number of sample observations

Point and Interval Estimates for Mean – Population Standard deviation (σ) known - Examples

1. The width of the interval is determined by the level of confidence and the size of the standard error of the mean.
2. The standard error is affected by two values:
   - Standard deviation
   - Number of observations in the sample

Point and Interval Estimates for Mean – Population Standard deviation (σ) known - Examples

Ex. The American Management Association wishes to have information on the mean income of middle managers in the retail industry. A random sample of 256 managers reveals a sample mean of $45,420. The standard deviation of this population is $2,050. The association would like answers to the following questions:

1. What is the population mean?
2. What is a reasonable range of values for the population mean?
3. What do these results mean?

Point and Interval Estimates for Mean – Population Standard deviation (σ) known - Examples

1. What is the population mean?

- **Solution:** In this case, we do not know population mean.

- We do know the sample mean which is $45,420 (given in question).
- Hence, our best estimate of the unknown population value is the corresponding sample statistic.
- The sample mean of $45,420 is a ***point estimate*** of the unknown population mean.

Point and Interval Estimates for Mean – Population Standard deviation (σ) known - Examples

2. What is a reasonable range of values for the population mean?

Solution:

Suppose the association decides to use the 95 percent level of confidence:

$$= \overline{X} \pm z\frac{\sigma}{\sqrt{n}}$$

$$= \$45420 \pm 1.96 \pm \frac{\$2050}{\sqrt{256}}$$

$$= \$45420 \pm 196\frac{2050}{16}$$

$$= \$45420 \pm 1.96(\$128.125)$$

Point and Interval Estimates for Mean – Population Standard deviation (σ) known - Examples

$= \$45420 \pm \$251$

$= \$45420 + \$251 \; and = \$45420 - \$251$

Confidence Limit = \$45169 to \$45671

The ±\$251 is referred to as the margin of error.

Point and Interval Estimates for Mean – Population Standard deviation (σ) known - Examples

3. What do these results mean, i.e. what is the interpretation of the confidence limits $45,169 and $45,671?

Solution: If we select many samples of 256 managers, and for each sample we compute the mean and then construct a 95 percent confidence interval, we could expect about 95 percent of these confidence intervals to contain the *population* mean. Conversely, about 5 percent of the intervals would not contain the population mean annual income, μ.

We could expect 95 times out of 100 that the mean will be within this limit. Only 5 out of 100 times, mean may be outside the limit.

Point and Interval Estimates for Mean – Population Standard deviation (σ) known - Examples

THANK YOU