* Secant Method:-

Newton-Raphson method requires the evaluation of derivatives of the function and this is not always possible, particularly in the case of functions arising in practical problems.

In Secant method, the derivative at x_i is approximated by the formula

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

which can be written as

$$f_i' = \frac{f_i - f_{i-1}}{x_i - x_{i-1}}$$

where $f_i = f(x_i)$.

Hence the Newton-Raphson formula becomes

$$x_{i+1} = x_i - \frac{f_i(x_i - x_{i-1})}{f_i - f_{i-1}} = \frac{x_{i-1} f_i - x_i f_{i-1}}{f_i - f_{i-1}}$$

It should be noted that this formula requires two initial approximations to the root.

Example Find the real root of the equation $x^3 - 2x - 5 = 0$ using secant method.

Solution Given equation -

$$x^3 - 2x - 5 = 0$$

Let the two initial approximations be given by $x_{-1} = 2$ and $x_0 = 3$

We have

$$f_{-1} = f(x_{-1}) = 2^3 - 2\times 2 - 5 = -1$$

$$f_0 = f(x_0) = 3^3 - 2\times 3 - 5 = 16$$

Putting $i=0$ in Eqn of Secant method.

$$x_{i+1} = \frac{x_{i-1} f_i - x_i f_{i-1}}{f_i - f_{i-1}} \quad \text{--- (1)}$$

Then

$$x_1 = \frac{x_{-1} f_0 - x_0 f_{-1}}{f_0 - f_{-1}} = \frac{2\times 16 - 3\times(-1)}{16-(-1)}$$

$$x_1 = \frac{32+3}{17} = \frac{35}{17} = 2.058823529$$

Also, $f(x_1) = f_1 = (2.058823529)^3 - 2\times(2.058823529) - 5$

$$f_1 = -.390799923$$

Putting $i=1$ in Eqn (1) we obtain.

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

Then

$$x_2 = \frac{3(-0.390799923) - 2.058823529 \times 16}{-16.390799923}$$

$$x_2 = 2.08126366$$

Again

$$f(x_2) = f_2 = (2.08126366)^3 - 2 \times (2.08126366) - 5$$

$$f_2 = -0.147204057$$

and setting $i=2$ in Eqn (1) we obtain

$$x_3 = \frac{x_1 f_2 - x_2 f_1}{f_2 - f_1}$$

$$x_3 = \frac{2.058823529(-0.147204057) - (2.08126366) \times (-0.390799923)}{(-0.147204057) - (-0.390799923)}$$

$$x_3 = 2.094824145$$

which is correct to three significant figures.

$$x_3 = 2.0948 = 2.095$$

Ans